

## Programming \& Data Analysis with 'R’

Diarmuid O'Briain

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## Document Outline

This document brings together a collection of different $R$ experiences. It documents work carried out during a week long training on "Statistical Data Analysis using R-software" that took place from 24-28 September 2018 at the Directorate of Research and Graduate Training (DRGT), Makerere University, Kampala, Uganda that was delivered by Associate Professor Matthew Low and Dr. Matthew Hiron of the Swedish University of Agricultural Sciences (SLU) to develop the programming aspect of $R$ with a focus on the analysis of quantitative data sets. I have included a section on qualitative data analysis as $R$ has some tools in that area too.
The document explores the fundamentals of how to use $R$. Installing $R$, the structure of $R$, vectors, matrices, arrays, lists, tables, data-frames as well as flow control and user-defined functions. Working with quantitative data, linear models, predictions, probability distributions, distribution models and generalised linear mixed models. Plotting data for visual output. Finally both inductive and deductive approaches to the analysis of qualitative data is explored.

## Diarmuid Ó Briain

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## Table of Abbreviations

| AIC | Akaike Information Criterion |
| :--- | :--- |
| ANCOVA | Analysis of covariance |
| ANOVA | Analysis of Variance |
| CO $_{2}$ | Oxygen |
| CRAN | Comprehensive R Archive Network |
| CSS | Cascading Style Sheets |
| CSV | Comma Separated file |
| DRGT | Directorate of Research and Graduate Training |
| DV | Dependent Variable |
| FUN | Function |
| GLM | Generalised Linear Model |
| GLMM | Generalised Linear Mixed Model |
| GUI | Graphical User Interface |
| HTML | Hypertext Markup Language |
| IV | Independent Variable |
| LMM | Linear Mixed-effects Model |
| MANOVA | Multivarite ANOVA |
| NLME | Non-linear Mixed-Effects Model |
| p-test | Statistical method used to assess one or more hypotheses within a population |
| R | or a proportion within a population |
| REML | Open-source free statistical programming language used by scientists |
| RQDA | Restricted (or Residual, or Reduced) Maximum Likelihood |
| RD Qualitative Data Analysis |  |
| SE | Standard Deviation |
| SLU | Standard Errors |
| SQL | Swedish University of Agricultural Sciences |
| Student t-test | Structured Query Language introduced by William Sealy Gosset |
| SVG | Scalable Vector Graphics |
| t-test | Analysis of two populations means through the use of statistical examination |
| Tukey | Tukey's Honest Significant Difference |
| Welch t-test | Adaptation of Student's t-test |
| YAML | YAML Ain't Markup Language |

## 1. Installing $\mathbf{R}$

This document is based on implementation on the GNU/Linux operating system. It assumes a Debian GNU/Linux based distribution like Debian, Ubuntu, Linux Mint, Elementary OS or a host of others. If the GNU/Linux operating system is RedHat or Fedora based like CentOS then the yum package manager is used. Replace the sudo apt-get install <package name> with sudo yum install <package name>. For other operating systems like Apple macOS or https://cloud.r-project.org/bin/windows/ Microsoft please consult the relevant $r$-project links.
$R$ can be used directly from the terminal shell and text editor tools like xed and kate have built in highlight modes, typically under the Scientific sections of these programs. Alternatively install an Integrated Development Environment (IDE) like Rstudio.
To operate with the terminal shell and text editors install the following base package.
\$ sudo apt-get install r-base
If Rstudio is required the also install the following packages.

```
$ sudo apt-get install rstudio
$ sudo apt-get install r-cran-rstudioapi
```

This document will proceed with the assumption of a terminal shell and a text editor.

## 2. Introduction

### 2.1 Simple R script

Starting out with a simple $R$ script and redirect it into the $R$ interpreter. Copy from cat to EOM on its own and paste to a terminal shell, best to do this in a directory specifically for the purpose. This creates a file called HelloWorld.R in the working directory. Redirect the file to the R interpreter.

```
$ cat << EOM >> HelloWorld.R
## run this code in R
## Hello World script
plot(x=1, y=1, typ="n", xlab="", ylab="", bty="n", xaxt="n", yaxt="n", ylim=c(-1,2))
text(x=1, y=1,"'R' is a language for statistical analysis", cex=2, col="red")
text(x=1, y=0.5, "R: Hello World", cex=3, col="blue")
EOM
$ R < test.R
```

A file called Rplots.pdf is generated in the working directory.

# ' $R$ ' is a language for statistical analysis 

## R: Hello World

Illustration 1: R: Hello World

### 2.2 Another simple script

Some basic maths in another script.

```
$ cat << EOM >> Introduction.R
# R introduction
1+3
4*7
a=3
a+7
EOM
$ R < Introduction.R --vanilla
R version 3.4.4 (2018-03-15) -- "Someone to Lean On"
Copyright (C) 2018 The R Foundation for Statistical Computing
Platform: x86_64-pc-linux-gnu (64-bit)
R is free software and comes with ABSOLUTELY NO WARRANTY.
You are welcome to redistribute it under certain conditions.
Type 'license()' or 'licence()' for distribution details.
    Natural language support but running in an English locale
R is a collaborative project with many contributors.
Type 'contributors()' for more information and
'citation()' on how to cite R or R packages in publications.
Type 'demo()' for some demos, 'help()' for on-line help, or
'help.start()' for an HTML browser interface to help.
Type 'q()' to quit R.
> # R introduction
> 1+3
[1] }
> 4*7
[1] 28
>a=3
> a+7
[1] 10
```


### 2.3 Start a new script

In both of these cases the script was redirected to the R interpreter. It is also possible to make the script executable and allow it find the interpreter by the inclusion of a shebang (\#!) line on the first line referring the operating system to the to the path of the $R$ interpreter. Make the file executable and run it.

```
$ cat << EOM >> vi my_first_script.R
#!/usr/bin/Rscript
# R introduction
1+3
4*7
a=3
a + 7
EOM
$ chmod +x my_first_script.R
$ ./my_first_script.R
[1] 4
[1] 28
[1] 10
```


### 2.4 Running an $R$ shell

An $R$ shell can be ran to execute a small number of commands. Many of the examples in this document are executed in this way.

```
$ R
R version 3.4.4 (2018-03-15) -- "Someone to Lean On"
Copyright (C) 2018 The R Foundation for Statistical Computing
Platform: x86_64-pc-linux-gnu (64-bit)
R is free software and comes with ABSOLUTELY NO WARRANTY.
You are welcome to redistribute it under certain conditions.
Type 'license()' or 'licence()' for distribution details.
    Natural language support but running in an English locale
R is a collaborative project with many contributors.
Type 'contributors()' for more information and
'citation()' on how to cite R or R packages in publications.
Type 'demo()' for some demos, 'help()' for on-line help, or
'help.start()' for an HTML browser interface to help.
Type 'q()' to quit R.
>
```

To run $R$ without the message each time add the --quiet option.

```
$ R --quiet
>
```


### 2.5 R errors

Consider the example below. As $b$ has not been defined $R$ has no way of carrying out the instruction and will therefore throw back an error as demonstrated.

```
$ R --quiet
> b + 7
Error: object 'b' not found
```


### 2.6 Commenting

Annotate code as you write, it makes it easier to read later. The annotations are preceded with a \# symbol. The Rscript interpreter will ignore the annotations.

```
$ cat << EOM >> vi my_first_annotate.R
#!/usr/bin/Rscript
a = list(c(1,2,3,4), # A vector
    "Some", # Add characters
    "Words", # Add a second set of characters
    c(5,6,7,8) # A second vector
    )
## Exercise
str (a) # Output the structure of 'a'
m = lapply(a[1], mean) # Apply the mean function to 1st list in 'a'
n = lapply(a[4], max) # Apply the max function to 4st list in 'a'
a$means = c(m,n) # Add a new item to the list
print(a)
EOM
$ chmod +x my_first_annotate.R
```

```
$ ./my_first_annotate.R
List of 4
    $ : num [1:4] 1 2 3 4
    $ : chr "Some"
    $ : chr "Words"
    $ : num [1:4] 5 6 7 8
[[1]]
[1] 1 2 3 4
[[2]]
[1] "Some"
[[3]]
[1] "Words"
[[4]]
[1] 5 6 7 8
```

\$means
\$means[[1]]
[1] 2.5
\$means[[2]]
[1] 8

### 2.6.1 Multiline commenting

Many programming languages offer the ability to do multiline commenting. This is useful if there are some lines in the script that you want the script to skip without the need to put \# before each line in the block. In this case use the structure below around the block. In this case the test is always FALSE and therefore the lines are skipped.

```
if (FALSE){
Lines to be ignored.
}
```

To demonstrate. Watch what happens if this structure is added to the script. The lines between 'if(FALSE)\{' and ' $\}$ ' are bypassed by the $R$ interpreter.

```
$ cat << EOM >> multiline_comment.R
#!/usr/bin/Rscript
a = list(c(1,2,3,4), # A vector
    "Some", # Add characters
    "Words", # Add a second set of characters
    c(5,6,7,8) # A second vector
    )
```

```
## Exercise
```


## Exercise

str (a) \# Output the structure of 'a'
str (a) \# Output the structure of 'a'
if(FALSE){
if(FALSE){
m = lapply(a[1], mean) \# Apply the mean function to 1st list in 'a'
m = lapply(a[1], mean) \# Apply the mean function to 1st list in 'a'
n = lapply(a[4], max) \# Apply the max function to 4st list in 'a'
n = lapply(a[4], max) \# Apply the max function to 4st list in 'a'
a$means = c(m,n) # Add a new item to the list
a$means = c(m,n) \# Add a new item to the list
}
}
print(a)
print(a)
EOM

```
EOM
```

```
$ R < my_first_annotate.R
List of 4
    $ : num [1:4] 1 2 3 4
    $ : chr "Some"
    $ : chr "Words"
    $ : num [1:4] 5 6 7 8
[[1]]
[1] 1 2 3 4
[[2]]
[1] "Some"
[[3]]
[1] "Words"
[[4]]
[1] 5 6 7 8
```

So what happened?

### 2.7 Listing the existing objects in $R$

Is(): Returns a vector of character strings giving the names of the objects in the specified environment.
objects(): Same as Is().

## $>\operatorname{ls}()$

| [1] "df" | "model1" | "model2" | "model3" | "model4" | "realmodel" |
| :--- | :--- | :--- | :--- | :--- | :--- |
| $[7]$ "v" | "v1" | "v2" | "v3" | "v4" | "x" |

### 2.7.1 Clearing the existing objects in $R$

rm(): Removes objects specified successively as character strings, or in a character vector list, or through a combination of both.
remove(): Same as $r m()$.
To clear the environment of objects, supply a list of objects to the $r m()$ or remove() functions.

```
> ls()
\begin{tabular}{llllll}
{\([1] ~ " d f " ~\)} & "model1" & "model2" & "model3" & "model4" & "realmodel" \\
[7] "v" & "v1" & "v2" & "v3" & "v4" & \(" x "\)
\end{tabular}
> rm(list=ls())
> ls()
character(0)
```

or alternatively:

```
> objects()
```

| [1] "df" | "model1" | "model2" | "model3" | "model4" |
| :--- | :--- | :--- | :--- | :--- |
| [7] "v" | "v1" | "v2" | "v3" | "v4" |

```
> remove(list=objects())
> objects()
character(0)
```


## 3. R structure

rnorm(): random generation for the normal distribution.
rnorm(5,mean=0,sd=1)

Note: $\operatorname{rnorm}(n$, mean $=, s d=$ ) is used to generate n normal random numbers with arguments mean and standard deviation.

```
> rnorm(5,mean=0,sd=1)
[1] 1.3136807 -1.2612950 0.9052489 0.8711972 -1.7449344
> plot(rnorm(5,mean=0,sd=1))
```



## function(function(object))

Note: The $R$ abs() method is one of the R mathematics functions, which is used to return the absolute positive value of an individual number, or an expression.
> abs(rnorm(5,mean=0,sd=1))
[1] 0.076097160 .577435381 .102992580 .424791721 .41277626

## function(function(function(object)))

$>\operatorname{mean}(\operatorname{abs}(\operatorname{rnorm}(100$, mean=0,sd=1)))
[1] 0.8621388

## 4. Getting help with functions

There are a number of helpful tools in $R$ to get more information.

- $\operatorname{args}()$ : - Lists the arguments of a function give as an argument.
- help(), ?: - Get the man page for the function given as an argument.
- example(): - Get examples of how the function given as an argument can be used.


### 4.1 Arguments of a function

To understand a new function and one needs the available arguments within the function.

## function(arguments)

```
> args(rnorm)
function (n, mean = 0, sd = 1)
NULL
```

- $\mathbf{n}$ - User specified, i.e. number of numbers
- mean - By default the mean = 0
- sd - Default standard deviation = 1

```
4.2 help()
> help() or
persp package:graphics R Documentation
Perspective Plots
Description:
    This function draws perspective plots of a surface over the x-y
    plane. 'persp' is a generic function.
Usage:
    persp(x, ...)
```


## 4.3 example()

Example function gives examples of the function given to it as an argument.

```
> example(mean)
mean> x <- c(0:10, 50)
mean> xm <- mean(x)
mean> c(xm, mean(x, trim = 0.10))
[1] 8.75 5.50
```

More information can be found online at:
quick-r
inside-r
r-graph gallery

## 5. R packages

### 5.1 The Comprehensive R Archive Network (CRAN)

The CRAN is a resource of packages that can be added to $R$.

## The Comprehensive R Archive Network website

### 5.2 Installing Packages in R

To install packages an internet connection is necessary.

```
> install.packages("lme4")
```

Once the package is downloaded it is there but the package must be loaded to use it.

```
> library(lme4)
```

In a script that has a dependency on a package it is helpful to have a line at the top of the file that causes the package to be installed if the package is not already installed. In the example, if the package Ime4 is not installed then $R$ will install it first before continuing with the script. If it is installed then the line is ignored. The library ('Ime4') line loads the library such that the functions within the package Ime4 become available within the program.

```
> if(!require(lme4)){install.packages("lme4")}
> library(lme4)
```


### 5.3 Check installed packages

Check packages already installed with the ip command.

|  | Package | LibPath | Version | Priority |
| :---: | :---: | :---: | :---: | :---: |
| base | "base" | "/usr/lib/R/library" | "3.4.4" | "base" |
| boot | "boot" | "/usr/lib/R/library" | "1.3-20" | "recommended" |
| class | "class" | "/usr/lib/R/library" | "7.3-14" | "recommended" |
| cluster | "cluste | "/usr/lib/R/library" | "2.0.6" | "recommended" |
| codetools | "codetools" | "/usr/lib/R/library" | "0.2-15" | "recommended" |
| compiler | "compiler" | "/usr/lib/R/library" | "3.4.4" | "base" |
| datasets | "datasets" | "/usr/lib/R/library" | "3.4.4" | "base" |
| foreign | "foreign" | "/usr/lib/R/library" | "0.8-70" | "recommended" |
| graphics | "graphics" | "/usr/lib/R/library" | "3.4.4" | "base" |
| grDevices | "grDevices" | "/usr/lib/R/library" | "3.4.4" | "base" |
| grid | "grid" | "/usr/lib/R/library" | "3.4.4" | "base" |
| KernSmooth | "KernSmooth" | "/usr/lib/R/library" | "2.23-15" | "recommended" |
| lattice | "lattice" | "/usr/lib/R/library" | "0.20-35" | "recommended" |
| MASS | "MASS" | "/usr/lib/R/library" | "7.3-49" | "recommended" |
| Matri | "Matrix" | "/usr/lib/R/library" | "1.2-12" | "recommended" |
| methods | "methods" | "/usr/lib/R/library" | "3.4.4" | "base" |
| mgcv | "mgcv" | "/usr/lib/R/library" | "1.8-23" | "recommended" |
| nlme | "nlme" | "/usr/lib/R/library" | "3.1-131" | "recommended" |
| nnet | "nnet" | "/usr/lib/R/library" | "7.3-12" | "recommended" |
| parallel | "parallel" | "/usr/lib/R/library" | "3.4.4" | "base" |
| rpart | "rpart" | "/usr/lib/R/library" | "4.1-13" | "recommended" |
| spatial | "spatial" | "/usr/lib/R/library" | "7.3-11" | "recommended" |
| splines | "splines" | "/usr/lib/R/library" | "3.4.4" | "base" |
| stats | "stats" | "/usr/lib/R/library" | "3.4.4" | "base" |
| stats4 | "stats4" | "/usr/lib/R/library" | "3.4.4" | "base" |
| survival | "survival" | "/usr/lib/R/library" | "2.41-3" | "recommended" |
| tcltk | "tcltk" | "/usr/lib/R/library" | "3.4.4" | "base" |

```
tools "tools" "/usr/lib/R/library" "3.4.4" "base"
utils "utils" "/usr/lib/R/library" "3.4.4" "base"
    Depends
base NA
boot "R (>= 3.0.0), graphics, stats"
class "R (>= 3.0.0), stats, utils"
cluster "R (>= 3.0.1)"
codetools "R (>= 2.1)"
compiler NA
datasets NA
foreign "R (>= 3.0.0)"
graphics NA
grDevices NA
grid NA
KernSmooth "R (>= 2.5.0), stats"
lattice "R (>= 3.0.0)"
MASS "R (>= 3.1.0), grDevices, graphics, stats, utils"
Matrix "R (>= 3.0.1)"
methods NA
mgcv "R (>= 2.14.0), nlme (>= 3.1-64)"
nlme "R (>= 3.0.2)"
nnet "R (>= 2.14.0), stats, utils"
parallel NA
rpart "R (>= 2.15.0), graphics, stats, grDevices"
spatial "R (>= 3.0.0), graphics, stats, utils"
splines NA
stats NA
stats4 NA
survival "R (>= 2.13.0)"
tcltk NA
tools NA
utils NA
Imports LinkingTo
base NA NA
boot NA NA
class "MASS" NA
cluster "graphics, grDevices, stats, utils" NA
codetools NA NA
compiler NA NA
datasets NA NA
foreign "methods, utils, stats" NA
graphics "grDevices" NA
grDevices NA NA
grid "grDevices, utils" NA
KernSmooth NA NA
lattice "grid, grDevices, graphics, stats, utils" NA
MASS "methods" NA
Matrix "methods, graphics, grid, stats, utils, lattice" NA
methods "utils, stats" NA
mgcv "methods, stats, graphics, Matrix" NA
nlme "graphics, stats, utils, lattice" NA
nnet NA NA
parallel "tools, compiler" NA
rpart NA NA
spatial NA NA
splines "graphics, stats" NA
stats "utils, grDevices, graphics" NA
stats4 "graphics, methods, stats" NA
survival "graphics, Matrix, methods, splines, stats, utils" NA
tcltk "utils" NA
tools NA NA
utils NA NA
    Suggests
base "methods"
boot "MASS, survival"
class NA
cluster "MASS"
```

```
codetools NA
compiler NA
datasets NA
foreign NA
graphics NA
grDevices "KernSmooth"
grid "lattice"
KernSmooth "MASS"
lattice "KernSmooth, MASS, latticeExtra"
MASS "lattice, nlme, nnet, survival"
Matrix "expm, MASS"
methods "codetools"
mgcv "splines, parallel, survival, MASS"
nlme "Hmisc, MASS"
nnet "MASS"
parallel "methods"
rpart "survival"
spatial "MASS"
splines "Matrix, methods"
stats "MASS, Matrix, SuppDists, methods, stats4"
stats4 NA
survival NA
tcltk NA
tools "codetools, methods, xml2, curl"
utils "methods, XML"
base NA "Part of R 3.4.4"
boot NA "Unlimited"
class NA "GPL-2 | GPL-3"
cluster NA "GPL (>= 2)"
codetools NA "GPL"
compiler NA "Part of R 3.4.4"
datasets NA "Part of R 3.4.4"
foreign NA "GPL (>= 2)"
graphics NA "Part of R 3.4.4"
grDevices NA "Part of R 3.4.4"
grid NA "Part of R 3.4.4"
KernSmooth NA "Unlimited"
lattice "chron" "GPL (>= 2)"
MASS NA "GPL-2 | GPL-3"
Matrix "MatrixModels, graph, SparseM, sfsmisc" "GPL (>= 2) | file LICENCE"
methods NA "Part of R 3.4.4"
mgcv NA "GPL (>= 2)"
nlme NA "GPL (>= 2) | file LICENCE"
nnet NA "GPL-2 | GPL-3"
parallel "snow, nws, Rmpi" "Part of R 3.4.4"
rpart NA "GPL-2 | GPL-3"
spatial NA "GPL-2 | GPL-3"
splines NA "Part of R 3.4.4"
stats NA "Part of R 3.4.4"
stats4 NA "Part of R 3.4.4"
survival NA "LGPL (>= 2)"
tcltk NA "Part of R 3.4.4"
tools NA "Part of R 3.4.4"
utils NA "Part of R 3.4.4"
License_is_FOSS License_restricts_use OS_type MD5sum
oot
boot NA
class NA
cluster NA
codetools NA
compiler NA
datasets NA
foreign NA
graphics NA
grDevices NA
grid NA NA NA NA
```

| KernSmooth | NA | NA | NA | NA |
| :---: | :---: | :---: | :---: | :---: |
| lattice | NA | NA | NA | NA |
| MASS | NA | NA | NA | NA |
| Matrix | NA | NA | NA | NA |
| methods | NA | NA | NA | NA |
| mgcv | NA | NA | NA | NA |
| nlme | NA | NA | NA | NA |
| nnet | NA | NA | NA | NA |
| parallel | NA | NA | NA | NA |
| rpart | NA | NA | NA | NA |
| spatial | NA | NA | NA | NA |
| splines | NA | NA | NA | NA |
| stats | NA | NA | NA | NA |
| stats4 | NA | NA | NA | NA |
| survival | NA | NA | NA | NA |
| tcltk | NA | NA | NA | NA |
| tools | NA | NA | NA | NA |
| utils | NA | NA | NA | NA |
|  | Needs | Built |  |  |
| base | NA | "3.4.4" |  |  |
| boot | "no" | "3.4.2" |  |  |
| class | "yes" | "3.4.2" |  |  |
| cluster | "yes" | "3.4.2" |  |  |
| codetools | "no" | "3.4.2" |  |  |
| compiler | NA | "3.4.4" |  |  |
| datasets | NA | "3.4.4" |  |  |
| foreign | "yes" | "3.4.4" |  |  |
| graphics | "yes" | "3.4.4" |  |  |
| grDevices | "yes" | "3.4.4" |  |  |
| grid | "yes" | "3.4.4" |  |  |
| KernSmooth | "yes" | "3.4.2" |  |  |
| lattice | "yes" | "3.4.2" |  |  |
| MASS | "yes" | "3.4.3" |  |  |
| Matrix | "yes" | "3.4.2" |  |  |
| methods | "yes" | "3.4.4" |  |  |
| mgcv | "yes" | "3.4.3" |  |  |
| nlme | "yes" | "3.4.2" |  |  |
| nnet | "yes" | "3.4.2" |  |  |
| parallel | "yes" | "3.4.4" |  |  |
| rpart | "yes" | "3.4.3" |  |  |
| spatial | "yes" | "3.4.2" |  |  |
| splines | "yes" | "3.4.4" |  |  |
| stats | "yes" | "3.4.4" |  |  |
| stats4 | NA | "3.4.4" |  |  |
| survival | "yes" | "3.4.2" |  |  |
| tcltk | "yes" | "3.4.4" |  |  |
| tools | "yes" | "3.4.4" |  |  |
| utils | "yes" | "3.4.4" |  |  |

## 6. Citing $\mathbf{R}$ in papers and publications

Credit where credit is due, when using $R$ or any of its packages it is important to credit the work of the developers. $R$ has a built-in function citation() to get the BibTex entry for reference management software or simply take the reference directly from the terminal as text.

### 6.1 Cite R

```
> citation()
    R Core Team (2018). R: A language and environment for statistical
    computing. R Foundation for Statistical Computing, Vienna, Austria.
    URL https://www.R-project.org/.
A BibTeX entry for LaTeX users is
    @Manual{,
        title = {R: A Language and Environment for Statistical Computing},
        author = {{R Core Team}},
        organization = {R Foundation for Statistical Computing},
        address = {Vienna, Austria},
        year = {2018},
        url = {https://www.R-project.org/},
    }
```

We have invested a lot of time and effort in creating $R$, please cite it
when using it for data analysis. See also 'citation("pkgname")' for
citing R packages.

### 6.2 Cite individual R packages

```
> citation(package = "lme4")
```

To cite lme4 in publications use:

```
    Douglas Bates, Martin Maechler, Ben Bolker, Steve Walker (2015).
    Fitting Linear Mixed-Effects Models Using lme4. Journal of
    Statistical Software, 67(1), 1-48. doi:10.18637/jss.v067.i01.
A BibTeX entry for LaTeX users is
    @Article{,
    title = {Fitting Linear Mixed-Effects Models Using {lme4}},
    author = {Douglas Bates and Martin M{\"a}chler and Ben Bolker and Steve Walker},
    journal = {Journal of Statistical Software},
    year = {2015},
    volume = {67},
    number = {1},
    pages = {1--48},
    doi = {10.18637/jss.v067.i01},
}
```


## 7. Vectors, the basic data structure in $\mathbf{R}$

The vector is the basic data structure in $R$. It contains element of the same type. The data types can be logical, integer, double, character, complex or raw. It is considered the fundamental data type in $R$.

- logical - TRUE and FALSE are reserved words denoting logical constants, whereas T and F are global variables whose initial values set to these.
- integer - Whole number (not a fraction) that can be positive, negative, or zero.
- double - Creates a double-precision vector of the specified length. The elements of the vector are all equal to 0 . It is identical to numeric.
- character - Type of indexing is useful when dealing with named vectors.
- complex - The vector can be specified either by giving its length, its real and imaginary parts, or modulus and argument.
- raw - Type is intended to hold raw bytes. It is possible to extract sub-sequences of bytes, and to replace elements.

```
>x = 3
> y = 5
> meatballs = 7
> meatballs + x - y
[1] 5
> 4 + 3
[1] 7
>a=4+3
> a
[1] 7
```

The vector is the fundamental data type in $R$.

```
e.g. [1] 12344567
```

Numbers being put together in a vector must be done in one of these four ways.

```
> x = c(1,2,3,4,5,6,7) # concatenate
> y = 1:7 # colon operator
> z = seq(1,7,1) # sequence, 1 to 7 in intervals of 2 i.e. 1 3 5 7
>m = rep(1:7,2) # repeat i.e. 1,2,3,4,5,6,7,1,2,3,4,5,6,7
```

Now consider each.

```
> x = c(1,2,3,4,5,6,7)
> y = 1:7
> z = seq(1,7,1)
> m = rep(1:7,2)
> x
[1] 1 2 3 4 5 6 7
y
[1] 1 2 3 4 5 6 7
z
[1] 1 2 3 4 5 6 7
> m
[1] 1234567
```

Try some variations.

```
> d = seq(1,12,3)
> d
[1] 1 4 4 7 10
>e = rep(1:4,3)
> e
    [1] 1 2 3 4 1 2 3 4 1 2 3 4
>f=rep(d,4)
> f
[1] 1
```

Note:

- c(...): - Combines or concatenates its arguments.
- seq(from, to, by= ): - Generate regular sequences.
- $\operatorname{rep}(x, \ldots)$ : replicates the values in ' $x$ '.

Combining these.

```
>x = 1:7
>y= seq(1,7,2)
>z = c(x,y)
2
[1] 11 2 3 4 5 6 7 1 3 5 7
```

Any time more than one number must be given to $R$, then it MUST be created using one of these functions.

### 7.1 Exercise: Generate vectors

```
5
1.0 1.3 1.6 1.9 2.2 2.5 2.8 3.1 3.4 3.7 4.0 4.3 4.6 4.9
```




```
576104 3 17
1434143414234
12341234123485
2.00 2.17 2.34 2.51 2.68 2.85 3.02 3.19 3.36 3.53 3.70 3.87 4.04 4.21 4.38 4.55 4.72
4.89 5.06 5.23 5.40 5.57 5.74 5.91 6.08
"bird" "cat" "ferret"
5 5 12 12 13 13 20 20
5
1.0 1.3 1.6 1.9 2.2 2.5 2.8 3.1 3.4 3.7 4.0 4.3 4.6 4.9
```

Answer:

```
5
> a = 5:23
1.0 1.3 1.6 1.9 2.2 2.5 2.8 3.1 3.4 3.7 4.0 4.3 4.6 4.9
> c = seq(1,4.9,0.3)
7 7 7 7 7 7 7 7 7 7 7 7 7 7 7 7 7 7
> c = rep(7, 18)
```



```
> c = seq(14,0,-2)
    57 6 10 4 3 17
> c = c(5,7,6,10,4,3,17)
1 2 3412 % 4 4 1 2 3 4
> rep(1:4,3)
1 2 3412 3 4 1 2 3 4 85
>c=c(rep(1:4,3), 85)
2.00 2.17 2.34 2.51 2.68 2.85 3.02 3.19 3.36 3.53 3.70 3.87 4.04 4.21 4.38 4.55 4.72
4.89 5.06 5.23 5.40 5.57 5.74 5.91 6.08
>c = seq(2,length.out=25,by=0.17)
"bird" "cat" "ferret"
> c = c("bird","cat","ferret")
5 5 12 12 13 13 20 20
>c = rep(c(5,12,13,20),each=2)
```


### 7.2 Length() of a vector

The length of a vector can be obtained by the length() function.

```
>c = (seq(1, 1000, 0.34))
> length(c)
[1] 2939
> c = length(seq(1, 1000, 0.34))
> c
[1] 2939
```


### 7.3 Vector principles

3 vector principles central to $R$ programming.

1. Recycling
2. Vectorisation
3. Filtering [indexing]

### 7.3.1 Recycling

When applying an operation to two vectors that requires them to be the same length, $R$ automatically recycles or repeats the shorter vector until it is the same length as the longer one.

```
> x = c(1,2,3)
>y = c(3,4,5)
> z = 15
>a=c(8,9)
>b = c(10, 20)
```

Looking at $x+y, 1+3=4,2+4=6$ and $3+5=8$.

$$
\begin{aligned}
& >x+y \\
& {[1] 468}
\end{aligned}
$$

Now looking at $x+a, 1+8=9,2+9=11$ and $3+$ ?. As a has ran out of elements the first element is recycled again so $3+8=11$. The following warning message is received.

```
> x+a
[1] 9 11 11
Warning message:
In x + a : longer object length is not a multiple of shorter object length
```

Now looking at $y^{*} b, 3^{*} 10=30,4^{*} 20=80$ and as a has ran out of elements the first is recycled again so $5^{*} 10=50$ and a warning message is received.

```
> y*b
[1] 30 80 50
Warning message:
In y * b : longer object length is not a multiple of shorter object length
```

And finally $y^{*} z, 3^{*} 15=45$ and as there are no more elements in $z, 15$ must be recycled and the final calculations are $4^{*} 15=60$ and $5 * 15=75$.

```
> y*z
[1] 45 60 75
Warning message:
In y * b : longer object length is not a multiple of shorter object length
```

The last example above is something that happens all the time, where all the elements in $y=c(3,4,5)$ are multiplied by the single element in $b=15$.

- $3 \times 15=45$
- $4 \times(15$ recycled $)=60$
- $5 x(15$ recycled $)=75$


### 7.3.2 Vectorisation

Scalars are vectors, therefore most functions that you can apply to a single value, you can apply to a vector of values.

```
> sqrt(16)
[1] }
> x = c(3,6,9,12,15)
> sqrt(x)
[1] 1.732051 2.449490 3.000000 3.464102 3.872983
> sqrt(x/2)
[1] 1.224745 1.732051 2.121320 2.449490 2.738613
> x^2
[1] 
> x/3
[1] 1 2 3 4 5
```

Note here the modulus.

```
> x %% 4 # modulus (x mod y) 5%%2 is 1
```

[1] 32103

Breaking this down further.

```
> 3 %% 4
[1] 3
> 6 %% 4
[1] 2
> 9 %% 4
[1] 1
> 12 %% 4
[1] 0
> 15 %% 4
[1] 3
```

Looking at a different modulus example, 4 and 5 are continuously recycled until all the elements of $x$ are calculated.

```
> x %% c(4,5)
[1] 3 1 1 2 3
Warning message:
In x%%c(4, 5) :
    longer object length is not a multiple of shorter object length
```


## Exercise: write code to produce this vector

```
1491625 36496481100
>a = seq(1,10,1)
> a^2
    [1] }1
> seq(1,10,1)^2
    [1]
> (1:10)^2
    [1]
```

Note the importance of the brackets. Leaving out the brackets gives a completely different answer.

| $>1: \mathbf{1 0 \wedge}^{\wedge}$ |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| $[1]$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 |
| $[19]$ | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 | 31 | 32 | 33 | 34 | 35 | 36 |
| $[37]$ | 37 | 38 | 39 | 40 | 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | 50 | 51 | 52 | 53 | 54 |
| $[55]$ | 55 | 56 | 57 | 58 | 59 | 60 | 61 | 62 | 63 | 64 | 65 | 66 | 67 | 68 | 69 | 70 | 71 | 72 |
| $[73]$ | 73 | 74 | 75 | 76 | 77 | 78 | 79 | 80 | 81 | 82 | 83 | 84 | 85 | 86 | 87 | 88 | 89 | 90 |
| $[91]$ | 91 | 92 | 93 | 94 | 95 | 96 | 97 | 98 | 99 | 100 |  |  |  |  |  |  |  |  |

### 7.3.3 Filtering [Indexing] of vectors

## vector1[vector2]

The elements of vector2 select the elements of vector1 via the [ ] brackets.

$$
>y=c(10,1,16,3,8,5,13,55,34,13)
$$

Access the third element of the concatenation $y$.
$>y[3]$
[1] 16

Now access the first four elements of the concatenation $y$.

```
> y[1:4]
[1] 101163
```

So what happens for $y[3,4,6,8]$ ?

```
>y[3,4,6,8]
Error in y[3, 4, 6, 8] : incorrect number of dimensions
```

Well $[3,4,6,8]$ is not a vector. Remember that a vector must be developed as concatenation, via a colon operator, a sequence or a repeat. So rewrite as:

```
> y[c(3,4,6,8)]
[1] 16 3 5 55
```

So what happened?, remember $y=c(10,1,16,3,8,5,13,55,34,13)$. The second vector selects elements from the first vector. 3 selects the third element of $y$ which is 16, 4 selects the forth element 3 , etc..

## Exercise (a)

Extract the 3rd, 4th and 7th numbers in this vector.
$y=c(1,1,2,3,5,8,13,21,34,55)$
$>y=c(1,1,2,3,5,8,13,21,34,55)$
$>x=c(3,4,7)$
> y [x]
[1] $2 \quad 313$
$>y[c(3,4,7)]$
[1] 2313

## Exercise (b)

Extract the first 6 numbers of the vector.
$>y=c(1,1,2,3,5,8,13,21,34,55)$
$>x=1: 6$
$>y[x]$
[1] 112358
> $y[1: 6]$
[1] 112358

## Exercise (c)

Extract the final number of this vector by using the length() function.

```
> y = c(1,1,2,3,5,8,13,21,34,55)
> y[length(y)]
[1] 55
```

Another way to achieve the same result is to:

[^0]
### 7.3.4 Other Filtering [Indexing] examples

| $>\min (\mathrm{y})$ | \# Returns the smallest element |
| :---: | :---: |
| [1] 2 |  |
| $>\max (\mathrm{y})$ | \# Returns the largest element |
| [1] 8 |  |
| > y[2:4] | \# Returns elements 2 - 4 |
| [1] 832 |  |
| > y[3:6] | \# Returns elements 3-6 |
| [1] 3245 |  |

### 7.3.5 Ordering elements in a vector

The order() function returns a permutation which rearranges its first argument into ascending or descending order, breaking ties by further arguments. As can be seen from the example below it returns the positions of the elements in $y$ based on the sequential size starting with position 2 which is 1 , then position 4 for the next lowest number 3 , etc..

```
> y = c(10,1,16,3,8,5,13,55,34,13)
> order(y)
    [1] 
```

If the actual ordered list of the elements instead of the relevant element positions is required then use the following.

```
> y[order(y)]
```

[1] $\begin{array}{lllllllllll} & 1 & 3 & 5 & 8 & 10 & 13 & 13 & 16 & 34 & 55\end{array}$

Reversing the order. The rev() function provides a reversed version of its argument or alternatively negate the $y$ in the order function.

```
> y[rev(order(y))]
    [1] 55 34 16 13 13 10
> y[order(-y)]
    [1] 55 34 16 13 13 10
```


### 7.3.6 Extracting elements from a vector

How do I extract numbers from a vector?

- Write down the name of the vector.
- Put square brackets after it.
- Put something inside the square brackets.

```
> y = c(10,1,16,3,8,5,13,55,34,13)
> y[5]
[1] 8
> y[7]
[1] 13
```

What if one wants to extract certain numbers?

All numbers $>7$.

```
> y = c(10,1,16,3,8,5,13,55,34,13)
> y[y< 7]
[1] 1 3 5
```

All the numbers from 1 to 6 .

```
> y[c(1,2,3,4,5,6)]
[1] 10 1 16 3 8 8
> y[1:6]
[1] 10 10 16 3 8 5
```

Only numbers equal to 3 .

```
> y = c(10,1,16,3,8,5,13,55,34,13)
> y[y = 3]
[1] 16
```

Using negatives to eliminate numbers from a list.

```
> y = c(10,1,16,3,8,5,13,55,34,13)
y[c(-2,-2)]
[1] 10 16 3
y[-1:-4]
[1] 8 5 13 55 34 13
```


### 7.4 Booleans

Returns True or False ( 1 or 0).

```
> y = c(1,1,2,3,5,8,13,21,34,55)
> y == 3
    [1] FALSE FALSE FALSE TRUE FALSE FALSE FALSE FALSE FALSE FALSE
y <= 13
    [1] TRUE TRUE TRUE TRUE TRUE TRUE TRUE FALSE FALSE FALSE
> y >= 8
    [1] FALSE FALSE FALSE FALSE FALSE TRUE TRUE TRUE TRUE TRUE
y > 8
    [1] FALSE FALSE FALSE FALSE FALSE FALSE TRUE TRUE TRUE TRUE
> y != 8
    [1] TRUE TRUE TRUE TRUE TRUE FALSE TRUE TRUE TRUE TRUE
> y^2 < 8
    [1] TRUE TRUE TRUE FALSE FALSE FALSE FALSE FALSE FALSE FALSE
```


### 7.4.1 Exercise

What happens for: mean $(y==3)$ ?
Note: Generic function for the (trimmed) arithmetic mean.
Well the mean of $y$, in other words sum(5)/length(5).

```
> mean(y)
[1] 14.3
> sum(y)/length(y)
[1] 14.3
```

' $==$ ' means exactly equal to. Remember the output of $y==3$ ?
> $y==3$
[1] FALSE FALSE FALSE TRUE FALSE FALSE FALSE FALSE FALSE FALSE
So that is $0,0,0,1,0,0,0,0,0,0$.
$>\operatorname{sum}(\mathrm{c}(0,0,0,1,0,0,0,0,0,0) / 10)$
[1] 0.1
It is the mean of the booleans returned from the logic $y==3$ statement or a single one in 10 elements.
$>$ mean $(y==3)$
[1] 0.1
Now put the logic boolean statements inside the square brackets.

- T (TRUE) = include
- $F($ FALSE $)=$ exclude
$>y=c(1,1,2,3,5,8,13,21,34,55)$
$>y[y<=10]$
[1] 112358
> $y[y==3]$
[1] 3
$>y[y<=10]$

[1] 112 | 1 |
| :--- |

$>y[y>=9]$
[1] 13213455
$>y[y>9]$
[1] 13213455
> y[y != 8]
[1] $\begin{array}{lllllllll}1 & 1 & 2 & 3 & 5 & 13 & 21 & 34 & 55\end{array}$
$>y\left[y^{\wedge} 2<10\right]$
[1] 1123
or map True (T) and False (F) to the values in the vector.

```
>a=c(1,3,5,7,9)
> a[c(T,F,F,T,F)]
[1] 1 7
```

In this case there are not enough boolean statements so they are recycled such that it is the same as: $a[c(T, F, T, F, T)]$.

```
> a[c(T,F)]
[1] 1 5 9
> a[c(T,F,T,F,T)]
[1] 1 5 9
```


### 7.4.2 Combining Booleans (and (\&), or (|) )

```
>x = c(3,6,9,12,15)
```

Both conditions must be True.
$>x>3 \& x<10$
[1] FALSE TRUE TRUE FALSE FALSE
Either condition is True.

```
> x == 12 | x < 5
```

[1] TRUE FALSE FALSE TRUE FALSE

### 7.4.3 Boolean operators

```
== equals
< is less than
> is greater than
<= less than or equal to
>= greater than or equal to
!= is not equal to
& and
| or
```


## Exercise 1

For the vector $f=c(1,2,3,6,10,15,21,25,29,30)$

1. Find all numbers equal to 15 .
2. Find all numbers greater than 9
3. Find all numbers not equal to 10
4. Find 6 th to 10 th vector elements
5. Find all except the final element
6. Find all numbers that are multiples of 5
7. Find all numbers less than or equal to 15
8. Find all numbers between 7 and 24

Answer:

```
# Find all numbers equal to 15.
f[f==15]
[1] }1
# Find all numbers greater than 9.
> f[f > 9]
[1]}1
# Find all numbers not equal to 10.
> f[f != 10]
[1] 1 
```

\# Find 6th to 10th vector elements.
$>f[f=6: 10]$
[1] $15 \begin{array}{lllll}15 & 25 & 29 & 30\end{array}$
\# Find all except the final element.
> f[-length(f)]
[1] $\begin{array}{lllllllllll}1 & 2 & 3 & 6 & 10 & 15 & 21 & 25 & 29\end{array}$
\# Find all numbers that are multiples of 5.
> f[(f \%\% 5) == 0]
[1] 10152530
\# Find all numbers less than or equal to 15.
$>f[f<=15]$
[1] $1 \begin{array}{llllll} & 2 & 3 & 6 & 10 & 15\end{array}$
\# Find all numbers between 7 and 24 .
> $f[(f>7) \&(f<24)]$
[1] $10 \begin{array}{lll}15 & 21\end{array}$

## Exercise 2

For the vector $f=c(1,2,3,6,10,15,21,25,29,30)$

1. Find all numbers greater than 9
2. Find all numbers not equal to 10
3. Find 6 th to 10 th vector elements
4. Find all except the final element
5. Find all numbers that are multiples of 5
6. Find all numbers less than or equal to 15
7. Find all numbers between 7 and 24

Answer:

```
> f = c(1, 2,3,6,10,15,21,25,29,30)
# Find all numbers greater than 9
> f[f > 9]
[1] 10
# Find all numbers not equal to 10
> f[f != 9]
    [1] 1 2 2 3 6 10 15 21 25 29 30
# Find 6th to 10th vector elements
> f[6:10]
[1] 15 21 25 29 30
# Find all except the final element
> f[-length(f)]
[1] 1
# Find all numbers that are multiples of 5
> f[f %% 5 == 0]
[1] 10 15 25 30
# Find all numbers less than or equal to 15
>f[f <= 15]
1] 1 2 3 6 10 15
# Find all numbers between 7 and 24
> f[(f > 7) & (f < 16)]
[1] 10 15
```


## 8. Building on Vectors, other data structures in $\mathbf{R}$

### 8.1 Basics of structures

vector[row]<br>matrix[row,column]<br>array[row,column,leve]]

### 8.1.1 Vectors

- The vector is the fundamental data type in $R$, scalars and matrices are just special types of vectors and that all things that apply to vectors, apply to these.


### 8.1.2 Matrix

- A matrix is a collection of data elements arranged in a two-dimensional rectangular layout. The following is an example of a matrix with 2 rows and 3 columns.
- The elements must be of the same type.
- Here is an example.

```
> a.matrix = matrix(c(2, 4, 3, 1, 5, 7), # the data elements
                                    nrow=2, # number of rows
                                    ncol=3, # number of columns
)
```

> a.matrix

|  | $[, 1]$ | $[, 2]$ | $[, 3]$ |
| :--- | ---: | ---: | ---: |
| $[1]$, | 2 | 3 | 5 |
| $[2]$, | 4 | 1 | 7 |


> a.matrix

|  | $[, 1]$ | $[, 2]$ | $[, 3]$ |
| :--- | ---: | ---: | ---: |
| $[1]$, | 2 | 4 | 3 |
| $[2]$, | 1 | 5 | 7 |

- Indexing Matrices


## matrix.name[Row,Column]

- So to access row 2 and column 3 - matrix.name[2,3] or to access all elements in row 2 - matrix.name[2,] or all the elements in column 3 - matrix.name[,3].

```
> a.matrix[2,3]
[1] 7
> a.matrix[2,]
[1] 157
```

```
> a.matrix[,3]
[1] 3 7
> a.matrix[1:2,2:3]
            [,1] [,2]
[1,] 4 3
[2,] 5 7
> a.matrix[-2,]
[1] 24 3
> a.matrix[,-2]
    [,1] [,2]
[1,] 2 3
[2,] 1 7
```

- How about finding all the rows where column 3 is greater than 6 ?

```
> a.matrix [a.matrix[,3] > 6,]
```

[1] 157

## Apply boolean questions to matrix

For a matrix of four rows and three columns with the data $1,5,9,2,6,10,3,7,11,4,8,12$ get the output of the following $R$ commands.

- z.matrix == 3
- z.matrix[,3] <= 10
- z.matrix != 3
- any(z.matrix[1,] == 3)
- any(z.matrix > 15)
- all(z.matrix ==3)
- all(z.matrix < 20)
- complete.cases(z.matrix)

```
> z.matrix = matrix(c(1,5,9,2,6,10,3,7,11,4,8,12),
                                nrow=4,
                                ncol=3,
                                byrow=TRUE
    )
```

> z.matrix
[,1] [,2] [,3]
$[1] \quad 1 \quad 5 \quad$,
$[2] \quad 2 \quad 6 \quad$,
$[3] \quad 3 \quad 7 \quad$,
$\begin{array}{llll}{[4,]} & 4 & 8 & 12\end{array}$
> z.matrix == 3
[,1] [,2] [,3]
$[1$,$] FALSE FALSE FALSE$
[2,] FALSE FALSE FALSE
[3,] TRUE FALSE FALSE
[4,] FALSE FALSE FALSE
> z.matrix[,3] <= 10
[1] TRUE TRUE FALSE FALSE

```
> z.matrix != 3
        [,1] [,2] [,3]
[1,] TRUE TRUE TRUE
[2,] TRUE TRUE TRUE
[3,] FALSE TRUE TRUE
[4,] TRUE TRUE TRUE
> any(z.matrix[1,] == 3)
[1] FALSE
> any(z.matrix > 15)
[1] FALSE
> all(z.matrix == 3)
[1] FALSE
> all(z.matrix < 20)
[1] TRUE
> complete.cases(z.matrix)
[1] TRUE TRUE TRUE TRUE
```


### 8.1.3 Array

Arrays are the R data objects which can store data in more than two dimensions. An array is created using the array() function. It takes vectors as input and uses the values in the dim (dimensions) parameter to create an array. The dim parameters define matrices of three rows, four columns and two deep, i.e. two matrices.

## array[row,column,leve]]

Here is an example.


Illustration 3: Array

```
> v1 = c(5,9,3)
> v2 = c(10,11,12,13,14,15)
> v3 = c(21,23,24,26,27,28,29)
> v4 = c(32,34,35,37,31,34,45,46)
> p = array(c(v1,v2,v3,v4),dim = c(3,4,2))
```

| > p |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| , , 1 |  |  |  |  |
|  | [,1] | [,2] | [,3] | [, 4] |
| [1, ] | 5 | 10 | 13 | 21 |
| [2, ] | 9 | 11 | 14 | 23 |
| [3, ] | 3 | 12 | 15 | 24 |
| , , 2 |  |  |  |  |
|  | [,1] | [, 2] | $[, 3]$ | [, 4] |
| [1, ] | 26 | 29 | 35 | 34 |
| [2, ] | 27 | 32 | 37 | 45 |
| [3, ] | 28 | 34 | 31 | 46 |

- Here is another example.

```
> v1 = c(5,9,3)
> v2 = c(10, 11, 12, 13, 14, 15)
> p = array(c(v1,v2),dim = c(3,3,2))
> p
, , 1
\begin{tabular}{lrrr} 
& {\([, 1]\)} & {\([, 2]\)} & {\([, 3]\)} \\
{\([1]\),} & 5 & 10 & 13 \\
{\([2]\),} & 9 & 11 & 14 \\
{\([3]\),} & 3 & 12 & 15
\end{tabular}
    , 2
\begin{tabular}{lrrr} 
& {\([, 1]\)} & {\([, 2]\)} & {\([, 3]\)} \\
{\([1]\),} & 5 & 10 & 13 \\
{\([2]\),} & 9 & 11 & 14 \\
{\([3]\),} & 3 & 12 & 15
\end{tabular}
```

Names can be given to the rows, columns and matrices in the array by using the dimnames parameter.

Here is example.

|  | COL1 | COL2 | COL3 |  |
| :---: | :---: | :---: | :---: | :---: |
| ROW1 | 5 | 10 |  |  |

Illustration 4: Array 2

```
>v1 = c(5,9,3)
> v2 = c(10,11, 12,13,14,15)
> column.names = c("COL1","COL2","COL3")
> row.names = c("ROW1","ROW2","ROW3")
> matrix.names = c("Matrix1","Matrix2")
> p = array(c(v1,v2),dim = c(3,3,2),dimnames =
list(row.names, column.names,matrix.names))
> p
, , Matrix1
\begin{tabular}{lrrr} 
& COL1 & COL2 & COL3 \\
ROW1 & 5 & 10 & 13 \\
ROW2 & 9 & 11 & 14 \\
ROW3 & 3 & 12 & 15
\end{tabular}
, , Matrix2
\begin{tabular}{lrrr} 
& COL1 & COL2 & COL3 \\
ROW1 & 5 & 10 & 13 \\
ROW2 & 9 & 11 & 14 \\
ROW3 & 3 & 12 & 15
\end{tabular}
```


## Exercise (a): Matrix

|  | $[, 1]$ | $[, 2]$ | $[, 3]$ | $[, 4]$ | $[, 5]$ |
| :--- | ---: | ---: | ---: | ---: | ---: |
| $[1]$, | 1 | 3 | 5 | 7 | 9 |
| $[2]$, | 11 | 13 | 15 | 17 | 19 |
| $[3]$, | 21 | 23 | 25 | 27 | 29 |
| $[4]$, | 31 | 33 | 35 | 37 | 39 |

Create a matrix like the one above, containing the sequence of odd numbers starting at 1 and counting up to fill the matrix.

## Answer:

```
> new.matrix = matrix(seq(1,length.out=20,by=2),
    nrow=4, ncol=5,
    byrow=TRUE
    )
```

> new.matrix

|  | $[, 1]$ | $[, 2]$ | $[, 3]$ | $[, 4]$ | $[, 5]$ |
| :--- | ---: | ---: | ---: | ---: | ---: |
| $[1]$, | 1 | 3 | 5 | 7 | 9 |
| $[2]$, | 11 | 13 | 15 | 17 | 19 |
| $[3]$, | 21 | 23 | 25 | 27 | 29 |
| $[4]$, | 31 | 33 | 35 | 37 | 39 |

## Exercise (b): Matrix

1. Create a matrix (called d1) with 6 rows and 4 columns (byrow=F) using the sequence of odd numbers starting at 1.
2. Extract the number from the 3rd column, 4th row and assign it to the variable name g1.
3. Extract the 6th row
4. Extract columns 2 \& 4 from d1 and call the new matrix d 2
5. Create a new matrix ( d 3 ), by doing something to d 1 , that contains the sequence of EVEN numbers starting at 2
6. Change d3 element 3rd row, 2nd column so that it equals 500 . Look at d3. Change 3rd row, 2nd column to Harry. Look at d3 again. Has anything changed? Why? Change Harry back to 500 . Now what has happened?
```
> d1 = matrix(seq(1,length.out=24,by=2),
    nrow=6, ncol=4,
    byrow=FALSE
    )
> d1
        [,1] [,2] [,3] [,4]
[1,] 1
[2,] 
[3,] 5
[4,] 
[5,] 
[6,] 11 23 35 47
> g1 = d1[4,3]
> g1
[1] 31
> d1[6,]
[1] 11 23 35 47
> c(d1[,2], d1[,4])
[1] 13 15 17 19 21 23 37 39 41 43 45 4
> d2 = matrix(c(d1[,2], d1[,4]),
    nrow=6,ncol=2,
    byrow=FALSE
        )
>d3 = d1 + 1
> d3
\begin{tabular}{lrrrr} 
& {\([, 1]\)} & {\([, 2]\)} & {\([, 3]\)} & {\([, 4]\)} \\
{\([1]\),} & 2 & 14 & 26 & 38 \\
{\([2]\),} & 4 & 16 & 28 & 40 \\
{\([3]\),} & 6 & 18 & 30 & 42 \\
{\([4]\),} & 8 & 20 & 32 & 44 \\
{\([5]\),} & 10 & 22 & 34 & 46 \\
{\([6]\),} & 12 & 24 & 36 & 48
\end{tabular}
> d3[3,2] = 500
```



All values became strings because in a matrix all values must be of the same type.

```
> d3[3,2] = 500
> d3
    [,1] [,2] [,3] [,4]
[1,] "2" "14" "26" "38"
[2,] "4" "16" "28" "40"
[3,] "6" "500" "30" "42"
[4,] "8" "20" "32" "44"
[5,] "10" "22" "34" "46"
[6,] "12" "24" "36" "48"
```

String type has been maintained.

## Exercise (c): Matrix filtering

```
> chick = matrix(c(seq(1,5,1),
    c(10, 15, 12, 13, 15, 8, 11, 9, 12, 13)),
    nrow=5, ncol=3,byrow=FALSE
    )
> colnames(chick) = c('Individual','Weight','Age')
> chick
        Individual Weight Age
\begin{tabular}{llrr}
{\([1]\),} & 1 & 10 & 8 \\
{\([2]\),} & 2 & 15 & 11 \\
{\([3]\),} & 3 & 12 & 9 \\
{\([4]\),} & 4 & 13 & 12 \\
{\([5]\),} & 5 & 15 & 13
\end{tabular}
```

From the chick matrix:

1. All rows where weight is less than 15 g .
2. Using the mean() function, calculate the mean age of chicks $>10 \mathrm{~g}$ in weight.
3. Add an extra column $(2,5,3,5,6)$ to the chick matrix using the cbind() function.
cbind(): - Take a sequence of vector, matrix or data-frame arguments and combine by columns or rows, respectively.

| > chick[chick[,2] < 15,] |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| Individual Weight Age |  |  |  |  |
| [1, ] | 1 | 10 | 8 |  |
| [2, ] | 3 | 12 | 9 |  |
| [3, ] | 4 | 13 | 12 |  |
| > mean(chick[chick[,2] > 10,'Age']) |  |  |  |  |
| [1] 11.25 |  |  |  |  |
| > chick $=$ cbind(chick, $\mathrm{c}(2,5,3,5,6)$ ) |  |  |  |  |
| > chick |  |  |  |  |
| Individual Weight Age |  |  |  |  |
| [1, ] | 1 | 10 | 8 | 2 |
| [2, ] | 2 | 15 | 11 | 5 |
| [3, ] | 3 | 12 | 9 | 3 |
| [4, ] | 4 | 13 | 12 | 5 |
| [5, ] | 5 | 15 | 13 | 6 |

### 8.1.4 The 'apply' family of functions

apply(): returns a vector or array or list of values obtained by applying a function to margins of an array or matrix.
apply(X, MARGIN, FUN, ...)

- $X$ : an array, including a matrix.
- MARGIN: 1 = rows, $2=$ columns, $\mathrm{c}(1,2)=$ both.
- FUN: the function.

```
> v1 = c(5,9,3)
> v2 = c(10,11,12,13,14,15,16,17,18)
> v3 = c(21,23,24, 26, 27,28,29,30)
> p = matrix(c(v1,v2,v3), nrow = 4, ncol = 5)
>p
\begin{tabular}{lrrrrr} 
& {\([, 1]\)} & {\([, 2]\)} & {\([, 3]\)} & {\([, 4]\)} & {\([, 5]\)} \\
{\([1]\),} & 5 & 11 & 15 & 21 & 27 \\
{\([2]\),} & 9 & 12 & 16 & 23 & 28 \\
{\([3]\),} & 3 & 13 & 17 & 24 & 29 \\
{\([4]\),} & 10 & 14 & 18 & 26 & 30
\end{tabular}
# Sum of each column
> apply(p, 2, sum)
[1] }2
# Get the mean of each row
> apply(p, 1, mean)
[1] 15.8 17.6 17.2 19.6
> apply(p, c(1,2), mean)
    [,1] [,2] [,3] [,4] [,5]
[1,] 
[2,] 
[3,]
[4,]
```

While it is possible to supply a vector $c(1,2)$ to the MARGIN argument it makes little sense with a matrix. However with an array it operates between the matrices within the array.

```
\(>\mathrm{v} 1=\mathrm{c}(5,9,3)\)
> \(\mathrm{v} 2=\mathrm{c}(10,11,12,13,14,15)\)
\(>\mathrm{v} 3=\mathrm{c}(21,23,24,26,27,28,29)\)
\(>\mathrm{v} 4=\mathrm{c}(32,34,35,37,31,34,45,46)\)
\(>\mathrm{a}=\operatorname{array}(\mathrm{c}(\mathrm{v} 1, \mathrm{v} 2, \mathrm{v} 3, \mathrm{v} 4), \operatorname{dim}=\mathrm{c}(3,4,2))\)
\(>\mathrm{a}\)
, , 1
```

| $[, 1]$ |  |  |  | $[, 2]$ |
| :--- | ---: | ---: | ---: | ---: |$[, 3] \quad[, 4]$

, , 2

|  | $[, 1]$ | $[, 2]$ | $[, 3]$ | $[, 4]$ |
| :--- | ---: | ---: | ---: | ---: |
| $[1]$, | 26 | 29 | 35 | 34 |
| $[2]$, | 27 | 32 | 37 | 45 |
| $[3]$, | 28 | 34 | 31 | 46 |

> apply(a, c(1,2), mean)
[,1] [,2] [,3] [,4]
[1,] 15.519 .524 .027 .5
[2,] 18.021 .525 .534 .0
$[3]$,

## Other apply() functions

Other apply() functions exists, some of which are described later.

- lapply(): For lists and data-frames.
- sapply(): A simplified wrapper function for lapply().
- sapply(): The multivariate apply which can vectorise arguments to a function that is not usually accepting vectors as arguments.

In short, mapply() applies a function to multiple list or multiple vector arguments.

### 8.2 Recap exercises

1. Create the following vectors:
```
3}6
33 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3
20 13 16 17 18 20
100
1
13}14141516 17 18 19 20 21 22 23 24 25 26 27 28 29
1
"bird" "fish" "cricket"
```

- Answer

```
> seq(3, 48, 3)
    [1] }\begin{array}{lllllllllllllllll}{3}&{6}&{9}&{12}&{15}&{18}&{21}&{24}&{27}&{30}&{33}&{36}&{39}&{42}&{45}&{48}
> rep(3,25)
    [1] 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3
> c(20,13, 16, 17, 18, 20)
[1] 20 13 16 17 18 20
> seq(100, 61, -3)
    [1] 100 97 94 91 88 [15 85 82 
> seq(1, 20, 1)
    [1] 11 2 2 3 4 4 5 5 6 % 7 8 8 9 10 11 12 13 14 15 16 17 18 19 20
> seq(13, 29, 1)
    [1] 13 14 15 16
> seq(1, 19, 2)
    [1] 1
> c('bird','fish','cricket')
[1] "bird" "fish" "cricket"
```

2. From the following vector

## 36912151821242730333639424548

- Multiply each element by 10.
- Take the square root of each element.
- Add three to each element.
- Standardise each element to the mean of the vector. (i.e. divide each element by the vector's mean).
- Multiply every second element by the vector's length.
- Answer

```
>d = seq(3, 48, 3)
> d
    [1] }
> d * 10
    [1]
> sqrt(d)
    [1] 1.732051 2.449490 3.000000 3.464102 3.872983 4.242641 4.582576
    [8] 4.898979 5.196152 5.477226 5.744563 6.000000 6.244998 6.480741
    [15] 6.708204 6.928203
d + 3
    [1] 6
> d/(mean(d))
    [1] 0.1176471 0.2352941 0.3529412 0.4705882 0.5882353 0.7058824
    [7] 0.8235294 0.9411765 1.0588235 1.1764706 1.2941176 1.4117647
    [15] 1.5294118 1.6470588 1.7647059 1.8823529
> d * c(1, length(d))
    [1] 
```

3. Using vector indexing extract the following vectors from the a vector where:
```
a = c(5, 1, 6, 10, 2, 7, 13, 12, 8)
516
5}55~513 5 5 1 6 10 2 7 13 12
125 7 8 10 12 13
numbers less than 10
numbers equal to 7
all even numbers
```

- Answer
> a[1:3]
[1] 516
> c(rep(a[1],3),a[7],a[1])
[1] $\begin{array}{lllll}5 & 5 & 5 & 13 & 5\end{array}$
> a[1:length(a) - 1]
[1] $\begin{array}{lllllllll}5 & 1 & 6 & 10 & 2 & 7 & 13 & 12\end{array}$
$>\mathrm{c}(\mathrm{a}[2], \mathrm{a}[5], \mathrm{a}[1], \mathrm{a}[3], \mathrm{a}[6], \mathrm{a}[9], \mathrm{a}[4], \mathrm{a}[8], \mathrm{a}[7])$
[1] $1 \begin{array}{lllllllll} & 2 & 5 & 6 & 7 & 8 & 10 & 12 & 13\end{array}$
$>\mathrm{a}[\mathrm{a}<10$ ]
[1] 516278
> $a[a==7]$
[1] 7
$>a[a \% \% 2==0]$
[1] $610 \quad 212 \quad 8$

4. Create the following matrix

|  | $[, 1]$ | $[, 2]$ | $[, 3]$ | $[, 4]$ |
| :--- | ---: | ---: | ---: | ---: |
| $[1]$, | 2 | 4 | 6 | 8 |
| $[2]$, | 10 | 12 | 14 | 16 |
| $[3]$, | 18 | 20 | 22 | 24 |
| $[4]$, | 26 | 28 | 30 | 32 |
| $[5]$, | 34 | 36 | 38 | 40 |
| $[6]$, | 42 | 44 | 46 | 48 |

- Extract the 3rd column.
- Extract the 5th row.
- Create a new matrix from the first three rows and last two columns.
- Extract all numbers greater than 25.
- Extract all numbers greater than 35 from row five.
- Extract all numbers greater than 35 from column four.
- Extract all numbers between 10 \& 30 from column one.
- Answer

```
> x.matrix = matrix(c(seq(2,48,2)),
    nrow=6, ncol=4, byrow=TRUE
    )
> x.matrix
\begin{tabular}{lrrrr} 
& {\([, 1]\)} & {\([, 2]\)} & {\([, 3]\)} & {\([, 4]\)} \\
{\([1]\),} & 2 & 4 & 6 & 8 \\
{\([2]\),} & 10 & 12 & 14 & 16 \\
{\([3]\),} & 18 & 20 & 22 & 24 \\
{\([4]\),} & 26 & 28 & 30 & 32 \\
{\([5]\),} & 34 & 36 & 38 & 40 \\
{\([6]\),} & 42 & 44 & 46 & 48
\end{tabular}
> x.matrix[,3]
[1] 6 14 22 30 38 46
> x.matrix[5,]
[1] 34 36 38 40
> x.matrix[1:3,3:4]
    [,1] [,2]
[1,] 6 8
[2,] 14 16
[3,] 22 24
> x.matrix[c(x.matrix[]) > 25]
    [1] 26 34 42 28 36 44 30 38 46 32 40 48
> x.matrix[5,][x.matrix[5,] > 35]
[1] 36 38 40
> x.matrix[,4][x.matrix[,4] > 35]
[1] 40 48
> x.matrix[,1][x.matrix[,1] > 10 & x.matrix[,1] < 30]
[1] 18 26
```

5. Create the following matrix

| individual |  |  |  |
| ---: | ---: | ---: | ---: |
| $[1]$, | 1 | 4 | 13 |
| $[2]$, | 2 | 7 | 15 |
| $[3]$, | 3 | 2 | 12 |
| $[4]$, | 4 | 9 | 14 |
| $[5]$, | 5 | 2 | 20 |

- Give the columns the names above: use colnames().
- Extract which individuals are older than five years old.
- What is the mean weight of all individuals.
- What is the mean weight of individuals older than five.
- Add a new row - i.e. c(6, 5, 15) - using rbind() function.
- Answer

```
> y.matrix = matrix(c(seq(1,5,1),c(4,7,2,9,2,13,15,12,14,20)),
    nrow=5,ncol=3, byrow=FALSE
    )
> colnames(y.matrix) = c('individual','weight','age')
> y.matrix
    individual weight age
\begin{tabular}{llll}
{\([1]\),} & 1 & 4 & 13 \\
{\([2]\),} & 2 & 7 & 15 \\
{\([3]\),} & 3 & 2 & 12 \\
{\([4]\),} & 4 & 9 & 14 \\
{\([5]\),} & 5 & 2 & 20
\end{tabular}
> y.matrix[y.matrix[,3] > 5,'individual']
[1] 12 3 4 5
or
> y.matrix[y.matrix[,3] > 5,1]
[1] 1 2 3 4 5
> mean(y.matrix[y.matrix[,1],2])
[1] 4.8
> mean(y.matrix[y.matrix[,3] > 5,2])
[1] }4.
> y.matrix = rbind(y.matrix, c(6,5,15))
> y.matrix
        individual weight age
\begin{tabular}{llll}
{\([1]\),} & 1 & 4 & 13 \\
{\([2]\),} & 2 & 7 & 15 \\
{\([3]\),} & 3 & 2 & 12 \\
{\([4]\),} & 4 & 9 & 14 \\
{\([5]\),} & 5 & 2 & 20 \\
{\([6]\),} & 6 & 5 & 15
\end{tabular}
```


### 8.3 Operators as functions

Operators can be used like functions. Place the values to supply to the operator function and the result will return as expected.

```
> '+'(1, 2)
[1] 3
> '='(x,2)
> x
[1] 2
> '<-'(y,3)
> y
[1] 3
```

Even arguments can be used as functions.

$$
>{ }^{\prime \wedge '}(y, 2)
$$

[1] 9

## 9. Output to standard out

## 9.1 print() and cat()

print(): prints its argument.
cat(): Concatenate and print. This function outputs the objects, concatenating the representations.
cat() performs much less conversion than print().

```
> string = 'My string'
> numbers = 1:12
> t_date = date()
> chain = c(string, numbers, t_date)
> cat(chain)
My string 1 2 3 4 5 6 7 8 9 10 11 12 Tue Sep 18 13:00:07 2018
> print(chain)
    [1] "My string" "1"
    [3] "2" "3"
    [5] "4" "5"
    [7] "6" "7"
    [9] "8" "9"
[11] "10" "11"
[13] "12" "Tue Sep 18 13:00:07 2018"
> cat(string, numbers, t_date)
My string 1 2 3 4 5 6 7 8 9 10 11 12 Tue Sep 18 13:00:07 2018
> print(string, numbers, t_date)
Error in print.default(string, numbers, t_date) :
    invalid 'quote' argument
> print(c(string, numbers, t_date))
[1] "My string" "1"
    [3] "2" "3"
    [5] "4" "5"
    [7] "6" "7"
    [9] "8" "9"
[11] "10" "11"
[13] "12" "Tue Sep 18 13:00:07 2018"
```


## 9.2 sprintf()

A wrapper for the C function sprintf, that returns a character vector containing a formatted combination of text and variable values. The format string is a character string, beginning and ending in its initial shift state, if any. The format string is composed of zero or more directives: ordinary characters (not \%), which are copied verbatim to the output stream; and conversion specifications, each of which results in fetching zero or more subsequent arguments. Each conversion specification is introduced by the character \%, and ends with a conversion specifier. In between there may be zero or more flags, an optional minimum field width, an optional precision and an optional length modifier.

| Notation | Description |
| :---: | :--- |
| \%s | a string |
| \%d | an integer |
| \%0xd | an integer padded with $x$ leading zeros |
| \%f | decimal notation with six decimals |
| \%xf | floating point number with $x$ digits after decimal point |
| \%e | compact scientific notation, e in the exponent |
| \%E | compact scientific notation, E in the exponent |
| \%g | compact decimal or scientific notation (with e ) |

## 9.3 paste() and paste0() functions

paste(): Concatenates vectors after converting to character with a single space as a separator.
paste0(): Concatenates vectors after converting to character with no separator.

- sep: The element which separates every term. It should be specified with character string format.
- collapse: The element which separates every result. It should be specified with character string format and it is optional.

The difference between paste() and paste $O_{()}$) is that the argument sep by default is " " (paste) and "" in (paste0).

```
> string = 'My string'
> numbers = 1:12
> t_date = date()
> chain = c(string, numbers, t_date)
> p_string = paste(string, numbers, t_date)
> chain
    [1] "My string" "1"
    [3] "2" "3"
    [5] "4" "5"
    [7] "6" "7"
    [9] "8" "9"
[11] "10" "11"
[13] "12" "Tue Sep 18 13:00:07 2018"
> p_chain
    [1] "My string 1 Tue Sep 18 13:00:07 2018"
    [2] "My string 2 Tue Sep 18 13:00:07 2018"
    [3] "My string 3 Tue Sep 18 13:00:07 2018"
    [4] "My string 4 Tue Sep 18 13:00:07 2018"
    [5] "My string 5 Tue Sep 18 13:00:07 2018"
    [6] "My string 6 Tue Sep 18 13:00:07 2018"
    [7] "My string 7 Tue Sep 18 13:00:07 2018"
    [8] "My string 8 Tue Sep 18 13:00:07 2018"
    [9] "My string 9 Tue Sep 18 13:00:07 2018"
[10] "My string 10 Tue Sep 18 13:00:07 2018"
[11] "My string 11 Tue Sep 18 13:00:07 2018"
[12] "My string 12 Tue Sep 18 13:00:07 2018"
```


### 9.4 Numbers

Double precision value, in fixed point decimal notation.

```
> number = 1234567.894567
> number1 = sprintf("%.2f", number)
> print(number1)
[1] "1234567.89"
> number1 = sprintf("%e", number)
> print(number1)
[1] "1234567.89"
# Exponential output
> number2 = sprintf("%e", number)
> print(number2)
[1] "1.234568e+06"
# Scientific notation
> number3 = sprintf("%a", number)
> print(number3)
[1] "0x1.2d687e50257c9p+20"
```

Note that with integers, if an non-integer is given $R$ will give an error. Either give the input as an integer. Optionally print using the floating point number with zero digits after the decimal point to get an integer from a non integer input.

```
# Integer
> number4 = sprintf("%d", number)
Error in sprintf("%d", 1234567.894567) :
    invalid format '%d'; use format %f, %e, %g or %a for numeric objects
> number1 = sprintf("%d", 1234567)
> print(number1)
[1] "1234567"
> number5 = sprintf("%.0f", number)
> print(number5)
[1] "1234568"
```


### 9.5 Character string

```
> word = "R programming"
> word1 = sprintf("%s is fun.", word)
> print(word1)
[1] "R programming is fun."
```


## 10. Flow control \& Data frames

## 10.1 if \& else conditionals

Used to create flexibility in programming.
Write an if() statement that tells if a number ( $x$ ) is positive (the console returns the word positive).

```
> x = 5
y = -4
> if (x > 0) print('positive') else print('negative')
[1] "positive"
> if (y > 0) print('positive') else print('negative')
[1] "negative"
> if (x > 0){
    print('positive')
}else{
    print('negative')
}
[1] "positive"
> if (y > 0){
    print('positive')
}else{
    print('negative')
}
[1] "negative"
```

- if \& if else are not vectorised, they only take single values.


## 10.2 ifelse conditional

ifelse(boolean condition,output if TRUE,output if FALSE)

```
> ifelse((x > 0),'positive','negative')
[1] "positive"
> ifelse((y > 0),'positive','negative')
[1] "negative"
```

ifelse is a vectorised function.

### 10.2.1 Exercise: ifelse

age $=\mathbf{c}(3,5,7,4,9,3,5,4,8,6)$

- Create a new variable (age.cat) where ages four and below are 0 and above four are 1 .

```
> age = c(3,5,7,4,9,3,5,4,8,6)
> age.cat = ifelse((age < 5),0,1)
> age.cat
[1] 0 1 1 0 1 0 1 0 1 1
```

- Create a new variable (age.limit) where ages six and above are included in a single age category of 6 .

```
> age.limit = ifelse((age < 6),age,6)
> age.limit
[1] 3 5 6 4 6 3 5 4 6 6
```

- Create a new variable (age.3cat) where ages are in three categories $1=(1$ to 4$)$, $2=(5$ to 6 ), and $3=(7$ to 9$)$

```
> age.3cat = ifelse(
    (age < 5),
        print ('1'),
        ifelse(
                (age > 4 & age < 7),
                print ('2'),
                ifelse(
                (age > 6),
                print ('3'),"NA"
            )
    )
)
```

[1] "1"
[1] "2"
[1] "3"

## $>$ age.3cat

[1] "1" "2" "3" "1" "3" "1" "2" "1" "3" "2"

### 10.3 Lists

- Lists are the $R$ objects which contain elements of different types like - numbers, strings, vectors and another list inside it. A list can also contain a matrix or a function as its elements. List is created using list() function.
- Here is an example.

```
> list_data = list("Orange", "Green", c(43,13,61), TRUE, 42.18, 218.2)
> list_data
[[1]]
[1] "Orange"
[[2]]
[1] "Green"
[[3]]
[1] 43 13 61
[[4]]
[1] TRUE
[[5]]
[1] 42.18
[[6]]
[1] 218.2
```

- Accessing elements in the list. Here accessing the forth element in the list.
> list_data[4]
[[1]]
[1] TRUE
- The $R$ List can combine objects of any mode into a single object.

```
> w = list(site=5,species=c(2,3,4,5,6,7,8,9),
    names=c('Captain Kirk','Richard Dawkins')
    )
```

- Use unclass() to view the list.

```
> unclass(w)
$site
[1] 5
$species
[1] 2 3 4 5 6 7 8 9
$names
[1] "Captain Kirk" "Richard Dawkins"
```

- Here are the various tools to extract from the list. These tools all index the second sub-vector of the list.

```
> w$site
[1] 5
> w$species
[1] 2 3 4 5 6 7 8 9
> w[['species']]
[1] 2 3 4 5 6 7 8 9
```

```
> w[[1]]
[1] 5
> w[3]
$names
[1] "Captain Kirk" "Richard Dawkins"
```

- Adding to the list.

```
> w$shopping.list = c('milk','eggs')
> w
$site
[1] 5
$species
[1] 2 3 4 5 6 7 8 9
$names
[1] "Captain Kirk" "Richard Dawkins"
$shopping.list
[1] "milk" "eggs"
```

To remove shopping.list from our list $w$.

```
> w$shopping.list = NULL
> w
$site
[1] 5
```

\$species
[1] 23456789
\$names
[1] "Captain Kirk" "Richard Dawkins"

Most statistical outputs (objects) are contained within an $R$ list.
$\operatorname{Im}()$ : - is used to fit linear models.
mtcars: - is a stored dataset.

```
> data(mtcars)
> a = lm(mpg~wt, data=mtcars)
> summary(a)
Call:
lm(formula = mpg ~ wt, data = mtcars)
Residuals:
Min 1Q Median 3Q Max
-4.5432-2.3647-0.1252 1.4096 6.8727
Coefficients:
    Estimate Std. Error t value Pr(>|t|)
(Intercept) 37.2851 1.8776 19.858 < 2e-16 ***
wt -5.3445 0.5591 -9.559 1.29e-10 ***
Signif. codes: 0 ،***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
Residual standard error: 3.046 on 30 degrees of freedom
Multiple R-squared: 0.7528, Adjusted R-squared: 0.7446
F-statistic: 91.38 on 1 and 30 DF, p-value: 1.294e-10
```

```
> unclass(a)
$coefficients
(Intercept) 
$residuals
        Mazda RX4
        -2.2826106
    Hornet Sportabout
            -0.2001440
            Merc 230
            2.3499593
            Merc 450SL
            -0.0502472
    Chrysler Imperial
            5.9810744
            Toyota Corona
            -2.6110037
    Pontiac Firebird
            2.4643670
        Ford Pantera L
            -4.5431513
        Mazda RX4 Wag
\begin{tabular}{rr} 
Datsun 710 & Hornet 4 Drive \\
-2.0859521 & 1.2973499 \\
Duster 360 & Merc 240D \\
-3.9053627 & 4.1637381 \\
Merc 280C & Merc 450SE \\
-1.1001440 & 0.8668731 \\
Cadillac Fleetwood Lincoln Continental \\
1.1733496 & 2.1032876 \\
Honda Civic & Toyota Corolla \\
1.7461954 & 6.4219792 \\
AMC Javelin & Camaro Z28 \\
-3.7268663 & -3.4623553 \\
Porsche 914-2 & Lotus Europa \\
0.1520430 & 1.2010593 \\
Maserati Bora & Volvo 142 E \\
-3.2053627 & -1.0274952
\end{tabular}
$effects
    (Intercept) wt
-113.6497374 -29.1157217
            -1.6613339
\begin{tabular}{rrr}
1.6313943 & 0.1111305 & -0.3840041 \\
0.6111305 & -0.7888695 & 1.1143917 \\
2.2137818 & 6.0995633 & 7.3094734 \\
-2.6694078 & -3.4150859 & -3.1915608 \\
1.7073457 & -4.2045529 & -2.4018616
\end{tabular}
$rank
[1] 2
$fitted.values
        Mazda RX4
        23.282611
Hornet Sportabout
        18.900144
                        Merc 230
        20.450041
            Merc 450SL
        17.350247
Chrysler Imperial
                8.718926
            Toyota Corona
        24.111004
    Pontiac Firebird
                16.735633
        Ford Pantera L
        20.343151
Mazda RX4 Wag
21.919770
Valiant
18.793255
Merc 280
18.900144
Merc 450 SLC
17.083024
Fiat 128
25.527289
Dodge Challenger
18.472586
Fiat X1-9
26.943574
Ferrari Dino
22.480940
\begin{tabular}{rr} 
Datsun 710 & Hornet 4 Drive \\
24.885952 & 20.102650 \\
Duster 360 & Merc 240 D \\
18.205363 & 20.236262 \\
Merc 280C & Merc 450SE \\
18.900144 & 15.533127 \\
Cadillac Fleetwood Lincoln Continental \\
9.226650 & 8.296712 \\
Honda Civic & Toyota Corolla \\
28.653805 & 27.478021 \\
AMC Javelin & Camaro Z28 \\
18.926866 & 16.762355 \\
Porsche 914-2 & Lotus Europa \\
25.847957 & 29.198941 \\
Maserati Bora & Volvo 142 E \\
18.205363 & 22.427495
\end{tabular}
```

\$assign
[1] 01
\$qr
\$qr
Mazda RX4

| (Intercept) | wt |
| ---: | ---: |
| -5.6568542 | -18.199514334 |
| 0.1767767 | 5.447820482 |
| 0.1767767 | 0.148230003 |


| Hornet 4 Drive | 0.1767767 | -0.016055881 |
| :--- | ---: | ---: |
| Hornet Sportabout | 0.1767767 | -0.057356801 |
| Valiant | 0.1767767 | -0.061027994 |
| Duster 360 | 0.1767767 | -0.081219555 |
| Merc 240D | 0.1767767 | -0.011466889 |
| Merc 230 | 0.1767767 | -0.004124504 |
| Merc 280 | 0.1767767 | -0.057356801 |
| Merc 280C | 0.1767767 | -0.057356801 |
| Merc 450SE | 0.1767767 | -0.172999378 |
| Merc 450SL | 0.1767767 | -0.110589098 |
| Merc 450SLC | 0.1767767 | -0.119767081 |
| Cadillac Fleetwood | 0.1767767 | -0.389599760 |
| Lincoln Continental | 0.1767767 | -0.421539139 |
| Chrysler Imperial | 0.1767767 | -0.407037927 |
| Fiat 128 | 0.1767767 | 0.170257160 |
| Honda Civic | 0.1767767 | 0.277639553 |
| Toyota Corolla | 0.1767767 | 0.237256431 |
| Toyota Corona | 0.1767767 | 0.121613854 |
| Dodge Challenger | 0.1767767 | -0.072041573 |
| AMC Javelin | 0.1767767 | -0.056439003 |
| Camaro Z28 | 0.1767767 | -0.130780659 |
| Pontiac Firebird | 0.1767767 | -0.131698458 |
| Fiat X1-9 | 0.1767767 | 0.218900467 |
| Porsche 914-2 | 0.1767767 | 0.181270739 |
| Lotus Europa | 0.1767767 | 0.296362637 |
| Ford Pantera L | 0.1767767 | -0.007795696 |
| Ferrari Dino | 0.1767767 | 0.065628162 |
| Maserati Bora | 0.1767767 | -0.081219555 |
| Volvo 142E | 0.1767767 | 0.063792566 |
| attr (, "assign") |  |  |
| [1] 0 1 |  |  |

\$qraux
[1] 1.1767771 .046354
\$pivot
[1] 12
\$tol
[1] 1e-07
\$rank
[1] 2
attr(,"class")
[1] "qr"
\$df.residual
[1] 30
\$xlevels
named list()
\$call
$\operatorname{lm}($ formula $=m p g \sim w t$, data $=$ mtcars $)$
\$terms
mpg ~ wt
attr(,"variables")
list(mpg, wt)
attr(,"factors")
wt
mpg 0
wt 1
attr(,"term.labels")
[1] "wt"
attr(,"order")

```
[1] 1
attr(,"intercept")
[1] 1
attr(,"response")
[1] 1
attr(,".Environment")
<environment: R_GlobalEnv>
attr(,"predvars")
list(mpg, wt)
attr(,"dataClasses")
    mpg wt
"numeric" "numeric"
```

\$model

|  | mpg | wt |
| :--- | ---: | ---: |
| Mazda RX4 | 21.0 | 2.620 |
| Mazda RX4 Wag | 21.0 | 2.875 |
| Datsun 710 | 22.8 | 2.320 |
| Hornet 4 Drive | 21.4 | 3.215 |
| Hornet Sportabout | 18.7 | 3.440 |
| Valiant | 18.1 | 3.460 |
| Duster 360 | 14.3 | 3.570 |
| Merc 240D | 24.4 | 3.190 |
| Merc 230 | 22.8 | 3.150 |
| Merc 280 | 19.2 | 3.440 |
| Merc 280C | 17.8 | 3.440 |
| Merc 450SE | 16.4 | 4.070 |
| Merc 450SL | 17.3 | 3.730 |
| Merc 450SLC | 15.2 | 3.780 |
| Cadillac Fleetwood | 10.4 | 5.250 |
| Lincoln Continental | 10.4 | 5.424 |
| Chrysler Imperial | 14.7 | 5.345 |
| Fiat 128 | 32.4 | 2.200 |
| Honda Civic | 30.4 | 1.615 |
| Toyota Corolla | 33.9 | 1.835 |
| Toyota Corona | 21.5 | 2.465 |
| Dodge Challenger | 15.5 | 3.520 |
| AMC Javelin | 15.2 | 3.435 |
| Camaro Z28 | 13.3 | 3.840 |
| Pontiac Firebird | 19.2 | 3.845 |
| Fiat X1-9 | 27.3 | 1.935 |
| Porsche 914-2 | 26.0 | 2.140 |
| Lotus Europa | 30.4 | 1.513 |
| Ford Pantera L | 15.8 | 3.170 |
| Ferrari Dino | 19.7 | 2.770 |
| Maserati Bora | 15.0 | 3.570 |
| Volvo 142E | 21.4 | 2.780 |

### 10.4 Table

- Tables return the number of each element in a vector. The vector contains one $a$, three $b$ and one $c$.

```
> vector = c("a", "a", "b", "b", "b", "c")
> table(vector)
vector
a b c
2 3 
> z = table(vector)
> z[2]
b
3
```


## 10.5 for() and while()

for(): is used to repeat a set of instructions, and it is used when you know in advance the values that the loop variable will have each time it goes through the loop.
while(): is be used to repeat a set of instructions, and it is often used when you do not know in advance how often the instructions will be executed.

```
> for (element in seq(0:10)){
    print(element);
}
[1] 1
[1] 2
[1] 3
[1] 4
[1] 5
[1] }
[1] 7
[1] 8
[1] 9
[1] }1
[1] }1
> numbers = c(1,2,4,8,16,32,64)
> for (loop in numbers){
    cat("Loop number: ",loop,"\n");
}
Loop number: 1
Loop number: 2
Loop number: 4
Loop number: 8
Loop number: 16
Loop number: 32
Loop number: 64
> cars = list('Ford', 'Saab', 'Toyota', 'Nissan', 'Volvo', 'Renault')
> position = 1
> for (loop in cars){
    cat("Car number: ",position, ':', loop,"\n");
        position = position + 1;
}
Car number: 1 : Ford
Car number: 2 : Saab
Car number: 3 : Toyota
```

```
Car number: 4 : Nissan
Car number: 5 : Volvo
Car number: 6 : Renault
> position = 1
> while(position < 5){
    print (cars[position]);
    position = position + 1;
}
[[1]]
[1] "Ford"
[[1]]
[1] "Saab"
[[1]]
[1] "Toyota"
[[1]]
[1] "Nissan"
```


### 10.6 Data-frame

A data frame is a table or a two-dimensional array-like structure in which each column contains values of one variable and each row contains one set of values from each column. A spreadsheet if you will. There MUST be the same number of values in each row and column. Cells cannot be empty and must al least have a logical constant which contains the missing value indicator $N A$.
Following are the characteristics of a data frame.

- The column names should be non-empty.
- The row names should be unique.
- The data stored in a data frame can be of numeric, factor or character type.
- Each column should contain same number of data items.

Here is an example.

```
> employee.data = data.frame(employee_id = c (1:5),
    employee_name = c("Áine", "Dónal", "Siobhán", "Sinéad", "Donnacha"),
    salary = c(623.3,515.2,611.0,729.0,843.25)
    start_date = as.Date(c("2012-01-01", "2013-09-23", "2014-11-15", "2014-05-11",
    "2015-03-27")),
    stringsAsFactors = FALSE
    )
```

- Print the data frame.

```
> employee.data
employee_id employee_name salary start_date
    Áine 623.30 2012-01-01
    Dónal 515.20 2013-09-23
                            Siobhán 611.00 2014-11-15
                                Sinéad 729.00 2014-05-11
    Donnacha 843.25 2015-03-27
```

- Access elements from the frame.

```
> data.frame(employee.data$employee_name,employee.data$start_date)
    employee.data.employee_name employee.data.start_date
1. Áine 2012-01-01
    Dónal 2013-09-23
    Siobhán 2014-11-15
    Sinéad 2014-05-11
    Donnacha 2015-03-27
    employee.data[1:2]
    employee_id employee_name
1 1 Áine
                    Dónal
                    Siobhán
                            Sinéad
Donnacha
```

- Extract first two rows.

```
> employee.data[1:2,]
    employee_id employee_name salary start_date
1 1 Áine 623.3 2012-01-01
2 2 Dónal 515.2 2013-09-23
```

- Consider the following $R$ dataframe.

```
> individual = c("a1", "a2", "a3", "a4")
> age = c(15,13,16,12)
> weight.class = c("high", "high", "low", "high")
> df = data.frame(individual, age, weight.class)
> df
    individual age weight.class
\begin{tabular}{llll}
1 & a1 & 15 & high \\
2 & a2 & 13 & high \\
3 & a3 & 16 & low
\end{tabular}
4 a4 12 high
```

- Use class() or str() to look at how each variable is classified.

```
> class(individual)
[1] "character"
> class(age)
[1] "numeric"
> class(weight.class)
[1] "character"
> class(df)
[1] "data.frame"
> str(individual)
chr [1:4] "a1" "a2" "a3" "a4"
> str(age)
num [1:4] 15 13 16 12
> str(weight.class)
chr [1:4] "high" "high" "low" "high"
```

```
> str(df)
'data.frame': 4 obs. of 3 variables:
$ individual : Factor w/ 4 levels "a1","a2","a3",..: 1 2 3 4
$ age: num 15 13 16 12
$ weight.class: Factor w/ 2 levels "high","low": 1 1 2 1
```


### 10.7 Indexing Data-frames

read.csv(): Imports data from a Comma Separated File (CSV) file.
head() and tail(): Return the first or last parts of a vector, matrix, table, data frame or function.

For demonstration purposes, import the bird_egg.csv file.

```
bird_egg = read.csv('Datasets/bird_egg.csv', header=TRUE)
> head(bird_egg)
    individual year clutch age eggs dist_food fail_fledge
                rm/bg
        rb/bkm 107 3
        bbk/bkm 108 3
        wbk/ym
    tail(bird_egg)
    individual year clutch age eggs dist_food fail_fledge
825 rr/jm 108 1 2 5 NA N
826 vm/bv 109 1 1 4 NA
827 m/wy2 108 1 3 5 NA N
828 hm/y 109 1
829 /hm 108 1
830 wm/yr 106 1 4 4 4 4 NA 
```

Reviewing the column for year is (Note: head() function used to limit the output to a useable output):
> head(bird_egg\$year)
[1] $\begin{array}{llllllll}101 & 97 & 107 & 108 & 106 & 103\end{array}$
> head(bird_egg[,2])
[1] $101 \quad 97 \quad 107 \quad 108 \quad 106 \quad 103$
The first five rows of age is either of these (Note: head() function used to limit the output to a useable output):

```
> bird_egg$age[1:5]
```

[1] 21272
> bird_egg[1:5,4]
[1] 21272

### 10.8 Add a new column

Add a column of squared values (e.g. age two ) to bird_egg?. It is a simple matter of naming it and assign values to it.

| > head(bird_egg) |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | ividual | year | clutch | age | eggs | dist_food | fail_fledge | age.square |
| 1 | rm/bg | 101 | 3 | 2 | 4 | 149 | 1 | 4 |
| 2 | wm/rb | 97 | 3 | 1 | 3 | 63 | 0 | 1 |
| 3 | rb/bkm | 107 | 3 | 2 | 4 | NA | 0 | 4 |
| 4 | bbk/bkm | 108 | 3 | 7 | 3 | NA | 0 | 49 |
| 5 | wbk/ym | 106 | 3 | 2 | 3 | NA | 1 | 4 |
| 6 | o/ym | 103 | 3 | 2 | 4 | 164 | $\bigcirc$ | 4 |

It is also possible to add to the data frame based on an ifelse() decision rule.
This example:

- Adds new column to the bird_egg data.frame and calls it egg.factor.
- Creates a 2 -level categorical variable for number of eggs.

| > head(bird_egg) |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| individual year c |  |  |  |  | dist_food fail_fledge age.square eggs.factor |  |  |  |  |
| 1 | rm/bg | 101 | 3 | 2 | 4 | 149 | 1 | 4 | few |
| 2 | wm/rb | 97 | 3 | 1 | 3 | 63 | 0 | 1 | many |
| 3 | rb/bkm | 107 | 3 | 2 | 4 | NA | 0 | 4 | few |
| 4 | bbk/bkm | 108 | 3 | 7 | 3 | NA | 0 | 49 | many |
| 5 | wbk/ym | 106 | 3 | 2 | 3 | NA | 1 | 4 | many |
| 6 | o/ym | 103 | 3 | 2 | 4 | 164 | 0 | 4 | few |

### 10.9 Single or double brackets for indexing?

Single and double square brackets. As is demonstrated below the single brackets extract a list whereas double brackets extract in a numeric vector style format.

```
> a = seq(1, 10, length.out = 10)
>b = seq(1, 10, length.out = 10)
> c = seq(1.75, 3.2, length.out = 10)
> d = seq(0.43, 1.6, length.out = 10)
> a = data.frame(w = a, x = b, y = c, z = d)
> m = a[1]
> n = a[[2]]
>o = a[3]
> p = a[[4]]
class(m)
[1] "data.frame"
> class(n)
[1] "numeric"
> class(o)
[1] "data.frame"
> class(p)
[1] "numeric"
```

```
> typeof(m)
[1] "list"
> typeof(n)
[1] "double"
> typeof(o)
[1] "list"
> typeof(p)
[1] "double"
> m
1
2 2
3 3
4 4
5 5
6 6
7 7
8
9 9
10 10
> n
    [1] 1
>0
1 1.750000
2 1.911111
3 2.072222
4 2.233333
5 2.394444
6 2.555556
7 2.716667
8 2.877778
9 3.038889
103.200000
> p
    [1] 0.43 0.56 0.69 0.82 0.95 1.08 1.21 1.34 1.47 1.60
```


### 10.10 Exercise: Data frame 1

Create a new third variable in the data frame (called age.cat) which is a categorical variable based on age where all individuals four or less are young and those greater than four are considered old.

- Answer:

```
 individual = 1:10
> age =c(3,5,7,4,9,3,5,4,8,6)
> age.df = data.frame (individual, age)
> age.df$age.cat = ifelse(age.df$age < 5, 'young', 'old')
> age.df
    individual age age.cat
1 1 3 young
2 5 old
    7 old
    young
        old
        young
        old
        young
        old
        old
```


### 10.11 Changing names within the data

It is often necessary to adjust the names within the data. Follow the example below.

```
> names(age.df)
[1] "Individual" "age" "age.cat"
names(age.df)[3] = "age2"
> age.df
    Individual age age2
1 3 young
-3 7 old
4 4 young
9 old
young
old
young
old
old
```


### 10.12 Read files into $R$

read.csv() and read.csv2():

- read.csv and read.csv2 are identical to read.table except for the defaults. They are intended for reading comma separated value files (.csv) or (read.csv2) the variant used in countries that use a comma as decimal point and a semicolon as field separator.


## read.table():

- Reads a file in table format and creates a data frame from it, with cases corresponding to lines and variables to fields in the file.
read.delim(): - This is a wrapper function for read.table() with default argument values that are convenient when reading in tab-separated data.


### 10.13 English / European

Many European computer settings are different to the original .csv delimited file settings because the comma is used to denote fractions of numbers (decimals).

- 3.14 (English setting)
- 3,14 (European setting)

Thus European computers use semi-colons in their comma separated files and commas for decimals. This causes great confusion to $R$ if import it the wrong way Open your csv file in a text editor and check how it looks.

- Use read.csv() for English settings
- Use read.csv2() for European settings


### 10.14 Set a working directory

Set a working directory or set file pathway to read in data files.

- setwd(): - Is used to set the working directory to dir.
- get.wd(): - Check what the working directory is set to.
- file.choose(): - Choose a File Interactively.

```
setwd('~/course_datasets')
ind = read.csv('individual.csv')
head(ind)
    individual trait
1 a 3
a a 4
3 g
4
6 g
```


### 10.15 Checks after importing Data-frame

The first steps after importing data.frame are:

1. Check it looks like it should - use head() \& tail().
2. Use summary() to get an overview of the data.
3. Use $\operatorname{str}($ ) to check the structure of each variable.
4. Fix variable classes if necessary with as.factor() or as.character(), as.numeric() or as.Date() functions.
5. Get the names of each variable using names() (tip: paste names in to your code for future reference).
6. Create the datasets you want to use for analysis by using data.frame indexing \& filtering.

### 10.16 Removing missing values from a data set

Complete cases is a logical (boolean) function that returns TRUE for each observation (vectors) or row (data frame) that is complete (i.e. has no missing value / NA) for a data.frame called data.

```
> complete.data = data[complete.cases(data),]
```


### 10.16.1 Example:

Use the dataset bird_egg.csv.

```
df = read.csv('bird_egg.csv')
> head(df)
    individual year clutch age eggs dist_food fail_fledge
rm/bg 101 
            wm/rb 
        bbk/bkm
            wbk/ym
                0/ym
> summary(df)
    individual year clutch age eggs
    rm/bg : 21 Min. : 97.0 Min. :1.000 Min. : 1.000 Min. :1.000
    bm/bw : 20 1st Qu.:102.0 1st Qu.:1.000 1st Qu.: 1.000 1st Qu.:4.000
    m/ro : 19 Median :105.0 Median :1.000 Median : 2.000 Median :4.000
    bb/m : 17 Mean :104.7 Mean :1.396 Mean : 2.667 Mean :4.032
    bb/rm : 16 3rd Qu.:107.0 3rd Qu.:2.000 3rd Qu.: 4.000 3rd Qu.:5.000
    g/m : 15 Max. :109.0 Max. :3.000 Max. :10.000 Max. :6.000
    (Other):722 NA's :1 NA's :5
        dist_food fail_fledge
    Min. : 8.00 Min. :0.0000
    1st Qu.: 10.00 1st Qu.:0.0000
    Median : 70.00 Median :1.0000
    Mean : 90.69 Mean :0.6679
    3rd Qu.:135.00 3rd Qu.:1.0000
    Max. :434.00 Max. :1.0000
    NA's :481 NA's :2
> str(df)
'data.frame': 830 obs. of 7 variables:
    $ individual : Factor w/ 252 levels "","/gm","/hm",..: 162 214 152 11 204 130 234
238 55 214 ...
    $ year : int 101 97 107 108 106 103 97 97 100 98 ...
    $ clutch : int 3 3 3 3 3 3 3 3 3 3 ...
    $ age : int 2 1 2 7 2 2 4 1 2 2 ...
    $ eggs : int 4 3 4 3 3 4 3 3 2 3 ...
    $ dist_food : int 149 63 NA NA NA 164 18 191 112 12 ...
    $ fail_fledge: int 1 0 0 0 1 0 0 0 0 0 ...
> df$year = as.factor(df$year)
> df$fail_fledge = as.factor(df$fail_fledge)
> df$clutch = as.factor(df$clutch)
> str(df)
    'data.frame': 830 obs. of 7 variables:
    $ individual : Factor w/ 252 levels "","/gm","/hm",..: 162 214 152 11 204 130 234
238 55 214 ...
    $ year : Factor w/ 13 levels "97","98","99",..: 5 1 11 12 10 7 1 1 4 2 ...
    $ clutch : Factor w/ 3 levels "1","2","3": 3 3 3 3 3 3 3 3 3 3 3 3 ...
    $ age : int 2 1 2 7 2 2 4 1 2 2 ...
    $ eggs : int 4 3 4 3 3 4 3 3 2 3 ...
    $ dist_food : int 149 63 NA NA NA 164 18 191 112 12 ...
    $ fail_fledge: Factor w/ 2 levels "0","1": 2 1 1 1 2 1 1 1 1 1 ...
```

$>$ names(df)
[1] "individual" "year" "clutch" "age" "eggs"
[6] "dist_food" "fail_fledge"

### 10.17 Data.frame object classes

| Integer | as.integer() |
| :--- | :--- |
| Numeric | as.numeric() |
| Character | as.character() |
| Factor | as.factor() |
| Date | as.Date() |
| Logical | as.logical() |

Examples to change object class.

```
> df\$clutch = as.factor(df\$clutch)
```

> df\$individual = as.character(df\$individual)
as.matrix() and as.data.frame() can be used for 2-dimensional objects.

### 10.18 Saving tables \& Data Frames

write.table(): - prints its required argument ' $x$ ' (after converting it to a data frame if it is not one nor a matrix) to a file or connection.
write.csv(): - Does the same as write.table() except in comma delimited format.
These will save the data-frame or matrix to a file in the current working directory (use get. wd() to check where that is and set. $w d($ () to change it.)

```
> write.csv(df.egg, "bird_egg2.csv")
```


### 10.19 subset() function

It is regular that a subset of a frame is necessary. The subset() function handles this event.

### 10.19.1 Columns subset



Obviously the same could be achieved by indexing.

$$
>d f . \operatorname{sub2}=\operatorname{df} . \operatorname{egg}[, c(1,3,4,5)]
$$

```
head(df.sub2)
    character factors age eggs
        rm/bg 3 2 4
        wm/rb }
        rb/bkm 3 2 4
        bbk/bkm 3
        wbk/ym 3}
            o/ym 3 2 4
```


### 10.20 Date and time

Date and time for timestamps or dates and time within datasets can be useful. First looking at getting the time and date from the system with the date() function. The example demonstrates getting the date and time plus formatting them using the format types documented in the table below.

```
> t = Sys.time()
> print(t)
[1] "2018-09-28 10:45:41 EAT"
> str(t)
POSIXct[1:1], format: "2018-09-28 10:45:41
> t.short = format( x = t, format = '%d %b %Y')
> print(t.short)
[1] "28 Sep 2018"
> str(t.short)
chr "28 Sep 2018"
```

Strings of data that are in character or other formats can be converted to a date format. First ensure the data is in character format, then apply the as.Date() function to change to a date format.

```
> c.date = 'Fri 02 Sep 2018 - 11:02:12'
> c.date = as.character('Fri 02 Sep 2018 - 11:02:12') # Must be 'chr' format
> str(c.date)
chr "Fri 02 Sep 2018 - 11:02:12"
> c.date = as.Date(t, format = '%a %d %b %Y - %X')
> print(c.date)
[1] "2018-09-28"
> str(c.date)
    Date[1:1], format: "2018-09-28"
> c.date.short = format( x = c.date, format = '%d %b %Y')
> print(c.date.short)
[1] "28 Sep 2018"
> str(c.date.short)
    chr "28 Sep 2018"
```

For more granularity with time there is a specific time() function that creates the vector of times at which a time series was sampled.

## Format types

\%a Locale's abbreviated weekday name.
\%A Locale's full weekday name.
\%b Locale's abbreviated month name.
\%B Locale's full month name.
\%c Locale's appropriate date and time representation.
\%C Century (a year divided by 100 and truncated to an integer) as a decimal number [00,99].
\%d Day of the month as a decimal number [01,31].
\%D Date in the format $\mathrm{mm} / \mathrm{dd} / \mathrm{yy}$.
Day of the month as a decimal number $[1,31]$ in a two-digit field with leading space character fill.
\%h A synonym for \%b.
$\%$ Hour (24-hour clock) as a decimal number [00,23].
$\% \quad$ Hour (12-hour clock) as a decimal number [01,12].
$\%$ Day of the year as a decimal number [001,366].
\%m Month as a decimal number [01,12].
$\% \mathrm{M}$ inute as a decimal number $[00,59]$.
\%n A <newline>.
$\%$ Locale's equivalent of either AM or PM.
12 -hour clock time $[01,12]$ using the AM/PM notation; in the POSIX locale, this shall be equivalent to \% : \%M : \%S \%p.
$\%$ Seconds as a decimal number [00,60].
\%t A <tab>.
\%T 24-hour clock time $[00,23]$ in the format HH:MM:SS.
\%u Weekday as a decimal number [1,7] (1=Monday).
Week of the year (Sunday as the first day of the week) as a decimal number
$\% \quad[00,53]$. All days in a new year preceding the first Sunday shall be considered to be in week 0 .
Week of the year (Monday as the first day of the week) as a decimal number $[01,53]$. If the week containing January 1 has four or more days in the new year, then it shall be considered week 1; otherwise, it shall be the last week of the previous year, and the next week shall be week 1.
$\%$ Weekday as a decimal number [ 0,6 ] ( $0=$ Sunday).
Week of the year (Monday as the first day of the week) as a decimal number
$\%$ W $[00,53]$. All days in a new year preceding the first Monday shall be considered to be in week 0 .
\%x Locale's appropriate date representation.
\%X Locale's appropriate time representation.
$\%$ Year within century [00,99].
$\%$ Y Year with century as a decimal number.
\%Z Timezone name, or no characters if no timezone is determinable.

### 10.21 lapply()

Another function from the apply() functions.
The list apply lapply() function operates like apply () with similar output. The lapply() function can operate on other objects like dataframes and lists. It returns a list of the same length as $X$, each element of which is the result of applying FUN to the corresponding element of $X$.

- lapply(): returns a vector or array or list of values obtained by applying a function to margins of an array or matrix.
- apply(X, FUN, ...)
- $X$ : an array, including a matrix.
- FUN: the function.

This example shows the mean() function applied to the elements of the vector at position three of the list.

```
> l = list("Orange", "Green", c(43,13,61), TRUE, 42.18, 218.2)
(1,"[", , 3)
> str(l)
List of 6
    $ : chr "Orange"
    $ : chr "Green"
    $ : num [1:3] 43 13 61
    $ : logi TRUE
    $ : num 42.2
    $ : num 218
> lapply(1[3],mean)
[[1]]
[1] 39
> df = data.frame(employee_id = c (1:5),
            employee_name = c("Áine","Dónal","Siobhán","Sinéad","Donnacha"),
            salary = c(623.3,515.2,611.0,729.0,843.25),
            start_date = as.Date(c("2012-01-01", "2013-09-23", "2014-11-15", "2014-05-
            11", "2015-03-27")),
            stringsAsFactors = FALSE
            )
    df
    employee_id employee_name salary start_date
1 1 Áine 623.30 2012-01-01
2 2 Dónal 515.20 2013-09-23
3 3 Siobhán 611.00 2014-11-15
4 4 Sinéad 729.00 2014-05-11
5 5 Donnacha 843.25 2015-03-27
> lapply(df[3],sum)
$salary
[1] 3321.75
```


### 10.22 User-defined Functions

One of the great strengths of any programming language is the ability to add functions, $R$ is no different. A function should be considered if there is any block of code that is being repeated in a script. Instead put the block of code in a function, feed the function values and return results.

```
# Defining function above main block of code
new_function_name = function(arg1, arg2, ... ){
    statements
    return(object)
}
# Call function from main block of code
new_function_name(arg1, arg2, ... )
```

The following script demonstrates how a user-defined function works. Create the my_function_demo. $R$ file, make it executable and run it. The steps breakdown:

1. The existing set of objects in $R$ are cleared.
2. Five vector objects are created and the last four are added to a dataframe.
3. The user-defined function is defined.
4. The main program follows:

- A loop of four sends the $v$ vector plus the each vector from the dataframe in turn as a list to the user-defined function.
- The function processes and returns the linear model to where it was called from.
- This response is assigned to the object model<loop \#>. In this way four models are defined.
- The model<loop \#> names are extracted from the list of objects.
- A loop through these models and their summary is output to standard out.

```
$ cat << EOM >> my_function_demo.R
#!/usr/bin/Rscript
# Clear objects from R
rm(list = ls())
# Define objects
v = 1:8
v1 = c(51,19,43,74,45, 26,83,42)
v2 = c(101,111,112,123,141,152,193,141)
v3 = c(214, 233,234, 226, 237, 248,269, 276)
v4 = c(322,354,385,377,381,314,425,416)
df = data.frame(v1,v2,v3,v4)
# Function 'realmod'
realmod = function(arg1, arg2){
    x = as.numeric(unlist(arg1));
    y = as.numeric(unlist(arg2));
    return(lm(y ~ x))
}
# Main program (calling the realmod function)
for (x in 1:4){
    assign(paste0('model',x), realmod(v,df[x]));
}
```

```
# Get list objects
obj_list = ls()
mod_list = grep ('model', obj_list)
# Loop through models and output to stdout
for (x in mod_list){
    cat(sprintf('%s',obj_list[x]),'\n');
    cat(strrep("=",6), "\n");
    print(summary(get(obj_list[x])))
}
# End script
quit(status = 0)
EOM
$ chmod +x my_function_demo.R
$ ./my_function_demo.R
model1
======
Call:
lm(formula = y ~ x)
Residuals:
Min 1Q Median 3Q Max
-25.036 -15.839 -2.821 14.670 29.857
Coefficients:
    Estimate Std. Error t value Pr(>|t|)
(Intercept) 38.393 17.764 2.161 0.0739.
x 2.107 3.518 0.599 0.5711
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
Residual standard error: 22.8 on 6 degrees of freedom
Multiple R-squared: 0.05643, Adjusted R-squared: -0.1008
F-statistic: 0.3588 on 1 and 6 DF, p-value: 0.5711
model2
======
Call:
lm(formula = y ~ x)
Residuals:
\begin{tabular}{rrrrr} 
Min & \(1 Q\) & Median & \(3 Q\) & Max \\
-27.750 & -6.607 & 1.321 & 2.107 & 34.107
\end{tabular}
Coefficients:
    Estimate Std. Error t value Pr(>|t|)
(Intercept) 89.893 14.384 6.250 0.000778 ***
x 9.857 2.848 3.461 0.013459 *
Signif. codes: 0 '***' 0.001 ،**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
Residual standard error: 18.46 on 6 degrees of freedom
Multiple R-squared: 0.6662, Adjusted R-squared: 0.6106
F-statistic: 11.98 on 1 and 6 DF, p-value: 0.01346
model3
======
Call:
lm(formula = y ~ x)
```

```
Residuals:
\begin{tabular}{rrrrr} 
Min & \(1 Q\) & Median & \(3 Q\) & Max \\
-12.155 & -6.801 & 1.726 & 6.319 & 10.726
\end{tabular}
Coefficients
                Estimate Std. Error t value Pr(>|t|)
(Intercept) 206.393 6.989 29.531 1e-07 ***
X 7.940 1.384 5.737 0.00122 **
Signif. codes: 0 ،***' 0.001 ،**' 0.01 ،*' 0.05 '.' 0.1 ، ' 1
Residual standard error: 8.97 on 6 degrees of freedom
Multiple R-squared: 0.8458, Adjusted R-squared: 0.8201
F-statistic: 32.91 on 1 and 6 DF, p-value: 0.001218
model4
======
Call:
lm(formula = y ~ x)
Residuals:
\begin{tabular}{lrrrr} 
Min & \(1 Q\) & Median & \(3 Q\) & Max
\end{tabular}
-72.107 -0.714 8.107 14.964 29.321
Coefficients:
Estimate Std. Error t value Pr(>|t|)
(Intercept) 328.679 27.290 12.044 1.99e-05 ***
x 9.571 5.404 1.771 0.127
Signif. codes: 0 ،***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
Residual standard error: 35.02 on 6 degrees of freedom
Multiple R-squared: 0.3433, Adjusted R-squared: 0.2339
F-statistic: 3.137 on 1 and 6 DF, p-value: 0.1269
```


### 10.23 Recap exercises

### 10.23.1 Exercise: Data-frames 2

1. Import the bird egg.csv data and call the data frame df.egg.
2. Explore the data frame and check structure - change year, fail_fledge and clutch to factors, \& individual to character.
3. Create new column in df.egg called constant consisting of a column of 1 s .
4. Create a new data frame (by indexing) consisting of individual, clutch, age \& egg and call it df.sub1.
5. Create a new data.frame (called df.sub2) which consists of all data from first clutches for birds less than three years old.
6. Create a new data.frame (called df.sub3) which contains all data for which there are no missing values...

- HINT: complete.cases().

7. Calculate the mean distance to food for failed versus fledged nests. Do this for all data.frames (df.sub1, 2 \& 3)....

- HINT: check help(mean) if having problems.

8. Create a new column (dist.cat) that is a three-level distance category based on the dist_food column. Where <100 is near, 100-200 is mid, and >200 is far.

- HINT: ifelse(... , ... , ifelse(... , ... , ...)).

9. Create df.sub4 which contains all records without missing values in order of bird age (youngest to oldest)

- HINT: df.sub3[order(),].
10.Rename the individual column to id \& rename the dist_food column to distance.food.
11.Save the new data.frames as .csv files in your working directory folder (e.g. df.sub1.csv etc).

Answer:

```
##### // Read in the file //
> setwd('~/datasets/birds_egg')
> df.egg = read.csv('bird_egg.csv')
##### // Explore the data frame //
> summary(df.egg)
\begin{tabular}{|c|c|c|c|c|c|}
\hline \multicolumn{2}{|r|}{ndividual} & year & clutch & age & eggs \\
\hline rm/bg & 21 & Min. : 97.0 & Min. :1.000 & Min. : 1.000 & Min. 11.000 \\
\hline bm/bw & 20 & 1st Qu.:102.0 & 1st Qu.:1.000 & 1st Qu.: 1.000 & 1st Qu.:4.000 \\
\hline \(\mathrm{m} / \mathrm{ro}\) & 19 & Median :105.0 & Median :1.000 & Median : 2.000 & Median :4.000 \\
\hline \(\mathrm{bb} / \mathrm{m}\) & 17 & Mean :104.7 & Mean :1.396 & Mean : 2.667 & Mean :4.032 \\
\hline bb/rm & 16 & 3rd Qu.:107.0 & 3rd Qu.:2.000 & 3rd Qu.: 4.000 & 3rd Qu.:5.000 \\
\hline \(\mathrm{g} / \mathrm{m}\) & 15 & Max. :109.0 & Max. :3.000 & Max. :10.000 & Max. 66.000 \\
\hline (Othe & 22 & & & \(N A^{\prime}\) & NA's :5 \\
\hline
\end{tabular}
        dist_food fail_fledge
    Min. : 8.00 Min. :0.0000
    1st Qu.: 10.00 1st Qu.:0.0000
    Median : 70.00 Median :1.0000
    Mean : 90.69 Mean :0.6679
    3rd Qu.:135.00 3rd Qu.:1.0000
    Max. :434.00 Max. :1.0000
    NA's :481 NA's :2
> str(df.egg)
'data.frame': 830 obs. of 7 variables:
    $ individual : Factor w/ 252 levels "","/gm","/hm",..: 162 214 152 11 204 130 234
    238 55 214 ...
    $ year : int 101 97 107 108 106 103 97 97 100 98 ...
    $ clutch : int 3 3 3 3 3 3 3 3 3 3 ...
    $ age : int 2 1 2 7 2 2 4 1 2 2 ...
    $ eggs : int 4 3 4 3 3 4 3 3 2 3 ...
    $ dist_food : int 149 63 NA NA NA 164 18 191 112 12 ...
    $ fail_fledge: int 1 0 0 0 1 0 0 0 0 0 ...
> names(df.egg)
[1] "individual" "year" "clutch" "age" "eggs"
[6] "dist_food" "fail_fledge"
```

    head(df.egg)
    individual year clutch age eggs dist_food fail_fledge
        \(\begin{array}{lllllll}\mathrm{rm} / \mathrm{bg} & 101 & 3 & 2 & 4 & 149 & 1\end{array}\)
        wm/rb \(97 \begin{array}{llllll} & 3 & 1 & 3 & 63 & 0\end{array}\)
        \(\begin{array}{lllllll}\mathrm{rb} / \mathrm{bkm} & 107 & 3 & 2 & 4 & \text { NA }\end{array}\)
        \(\begin{array}{lllllll}\mathrm{bbk} / \mathrm{bkm} & 108 & 3 & 7 & 3 & \text { NA } & 0\end{array}\)
        \(\begin{array}{lllllrr}w b k / y m & 106 & 3 & 2 & 3 & \text { NA } & 1 \\ 103 & 3 & 2 & 4 & 164 & \end{array}\)
    \(\begin{array}{lllllll}0 / y m & 103 & 3 & 2 & 4 & 164 & 0\end{array}\)
    ```
##### // Change year, fail_fledge and clutch to factors //
> str(df.egg)
'data.frame': 830 obs. of 8 variables:
    $ individual : Factor w/ 252 levels "","/gm","/hm",..: 162 214 152 11 204 130 234
    238 55 214 ...
    $ year : int 101 97 107 108 106 103 97 97 100 98 ...
    $ clutch : int 3 3 3 3 3 3 3 3 3 3 ...
    $ age : int 2 1 2 7 2 2 4 1 2 2 ...
    $ eggs : int 4 3 4 3 3 4 3 3 2 3 ...
    $ dist_food : int 149 63 NA NA NA 164 18 191 112 12 ...
    $ fail_fledge: int 1000100 0 0 0 ...
> df.egg[, 'year'] = as.factor(df.egg[, 'year'])
> str(df.egg$year)
    Factor w/ 13 levels "97","98","99",..: 5 1 11 12 10 7 1 1 4 2 ...
> df.egg[, 'fail_fledge'] = as.factor(df.egg[, 'fail_fledge'])
> str(df.egg$fail_fledge)
    Factor w/ 2 levels "0","1": 2 1 1 1 2 1 1 1 1 1 ...
```

\#\#\#\#\# // Change individual to character //
> df.egg[, 'individual'] = as.character(df.egg[, 'individual'])
> str(df.egg\$individual)
chr [1:830] "rm/bg" "wm/rb" "rb/bkm" "bbk/bkm" "wbk/ym" "o/ym" "yg/m" ...
\#\#\#\#\# // Create new column in df.egg called 'constant' made of 1s //
df.egg\$constant = 1
> head(df.egg)
individual year clutch age eggs dist_food fail_fledge constant

| 1 | $\mathrm{rm} / \mathrm{bg}$ | 101 | 3 | 2 | 4 | 149 | 1 | 1 |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: | :--- | :--- |
| 2 | $\mathrm{wm} / \mathrm{rb}$ | 97 | 3 | 1 | 3 | 63 | 0 | 1 |
| 3 | $\mathrm{rb} / \mathrm{bkm}$ | 107 | 3 | 2 | 4 | $N A$ | 0 | 1 |
| 4 | $\mathrm{bbk} / \mathrm{bkm}$ | 108 | 3 | 7 | 3 | $N A$ | 0 | 1 |
| 5 | $\mathrm{wbk} / \mathrm{ym}$ | 106 | 3 | 2 | 3 | $N A$ | 1 | 1 |
| 6 | $0 / y m$ | 103 | 3 | 2 | 4 | 164 | 0 | 1 |

\#\#\#\#\# // Create a new data frame df.sub1 with individual, clutch, age \& eggs //
> names(df.egg)
$\begin{array}{ll}\text { [1] "individual" "year" } \\ \text { [6] "dist_food" "fail_fledge" "constant" } & \text { "age" }\end{array}$
$>$ df.sub1 $=d f . \operatorname{egg}[, c(1,3,4,5)]$
> head(df.sub1)
individual clutch age eggs

| 1 | $\mathrm{rm} / \mathrm{bg}$ | 3 | 2 | 4 |
| :--- | ---: | :--- | :--- | :--- |
| 2 | $\mathrm{wm} / \mathrm{rb}$ | 3 | 1 | 3 |
| 3 | $\mathrm{rb} / \mathrm{bkm}$ | 3 | 2 | 4 |
| 4 | $\mathrm{bbk} / \mathrm{bkm}$ | 3 | 7 | 3 |
| 5 | $\mathrm{wbk} / \mathrm{ym}$ | 3 | 2 | 3 |
| 6 | $0 / y m$ | 3 | 2 | 4 |

\#\#\#\#\# // df. sub2 to consists of all data for birds < 3 //
$>$ df.sub2 $=$ df.egg[df.egg\$age < 3,]

```
head(df.sub2)
    individual year clutch age eggs dist_food fail_fledge constant
\begin{tabular}{rrrrrrrr}
\(\mathrm{rm} / \mathrm{bg}\) & 101 & 3 & 2 & 4 & 149 & 1 & 1 \\
\(\mathrm{wm} / \mathrm{rb}\) & 97 & 3 & 1 & 3 & 63 & 0 & 1 \\
\(\mathrm{rb} / \mathrm{bkm}\) & 107 & 3 & 2 & 4 & \(N A\) & 0 & 1 \\
\(\mathrm{wbk} / \mathrm{ym}\) & 106 & 3 & 2 & 3 & \(N A\) & 1 & 1 \\
\(0 / y m\) & 103 & 3 & 2 & 4 & 164 & 0 & 1 \\
\(y m / b\) & 97 & 3 & 1 & 3 & 191 & 0 & 1
\end{tabular}
##### // df.sub3 to contain all data with no missing values //
> df.sub3 = df.egg[complete.cases(df.egg),]
> head(df.sub3)
    individual year clutch age eggs dist_food fail_fledge constant
\begin{tabular}{rrrrrrrr}
\(\mathrm{rm} / \mathrm{bg}\) & 101 & 3 & 2 & 4 & 149 & 1 & 1 \\
\(\mathrm{wm} / \mathrm{rb}\) & 97 & 3 & 1 & 3 & 63 & 0 & 1 \\
\(\mathrm{o} / \mathrm{ym}\) & 103 & 3 & 2 & 4 & 164 & 0 & 1 \\
\(\mathrm{yg} / \mathrm{m}\) & 97 & 3 & 4 & 3 & 18 & 0 & 1 \\
\(\mathrm{ym} / \mathrm{b}\) & 97 & 3 & 1 & 3 & 191 & 0 & 1 \\
\(\mathrm{bw} / \mathrm{m}\) & 100 & 3 & 2 & 2 & 112 & 0 & 1
\end{tabular}
##### // Calculate the mean distance to food for failed versus fledged nests //
Note: cannot do df.dub1 as it doesn't have a 'fail_fledge' column.
> mean(df.sub2[df.sub2$fail_fledge == 0 & complete.cases(df.sub2$dist_food),6])
[1] 118.2319
> mean(df.sub2[df.sub2$fail_fledge == 1 & complete.cases(df.sub2$dist_food),6])
[1] 93.60135
> mean(df.sub3[df.sub3$fail_fledge == 0 & complete.cases(df.sub3$dist_food),6])
[1] 96.70642
> mean(df.sub3[df.sub3$fail_fledge == 1 & complete.cases(df.sub3$dist_food),6])
[1] 87.9625
##### // 3-level distance category based on the 'dist_food' column //
> df.egg$dist.cat = ifelse(
            (df.egg$dist_food < 100),
            print ('near'),
            ifelse(
                (df.egg$dist_food > 100 & df.egg$dist_food < 200),
                print ('mid'),
                    ifelse(
                (df.egg$dist_food > 200),
                print ('far'),
            "NA"
            )
            )
            )
```

[1] "near"
[1] "mid"
[1] "far"


```
##### // df.sub4 contains all records without missing values in order of bird age //
    temp = df.egg[complete.cases(df.egg),]
    df.sub4 = temp[order(temp$age),]
> head(df.sub4)
    individual year clutch age eggs dist_food fail_fledge constant dist.cat
                wm/rb }070
                ym/b 
                m/ro 
                bg/m
                rm/bg 100 2rrrrrrrr
                wm/rb [lllllllll
##### // rename 'individual' --> 'id', 'dist_food' --> 'distance.food' //
> names(df.egg)
[1] "individual" "year" "clutch" "age" "eggs"
[6] "dist_food" "fail_fledge" "constant" "dist.cat"
> names(df.egg)[1] = "id"
> names(df.egg)[6] = "distance.food"
> names(df.egg)
[1] "id" \(\quad\) "year" \(\quad\) "distance.food" "fail_fledge" \(\quad\) "constant"
[9] "dist.cat"
> write.csv(df.sub1, file = "df.sub1.csv")
> write.csv(df.sub2, file = "df.sub2.csv")
> write.csv(df.sub3, file = "df.sub3.csv")
> write.csv(df.sub4, file = "df.sub4.csv")
```


### 10.23.2 Exercise: Data-frames 3

1. Create a new folder that contains the owl_data.csv data file.
2. Set your working directory to the new folder.
3. Import the owl_data.csv data and call the data frame owl.
4. Explore the data frame and check structure [e.g. by using summary(), str(), names(), head()].
5. Create a column of 1 's and 0 's called sex10 where male $=0$ \& female $=1$ (from the sex column).
6. Change the sex10 column to a factor

- HINT: as.factor().

7. Subset the data.frame to only include data from broods with five chicks.
8. Add 2 columns to the owl data.frame.

- food.category. where food is a 2-category variable low (food less than 25) \& high (food greater than or equal to 25).
- begging.3: where begging is a 3-category variable 1 (between $0-10$ ), 2 (from 10-20) and 3 (above 20).
- HINT: you can nest ifelse functions within ifelse functions.

9. Save the new data.frame as a .csv file called owl_2.csv.
```
##### // Inport the owl.csv file //
> setwd('~/datasets/owl_data')
> owl = read.csv('owl_data.csv')
##### // Explore the data frame //
> summary(owl)
    nest
\begin{tabular}{|c|c|c|c|c|c|}
\hline \multicolumn{2}{|r|}{nest} & ex & food & begging & brood \\
\hline Oleyes & 52 & Female:245 & Min. :21.71 & Min. : 0.00 & Min. \(: 1.000\) \\
\hline Moutet & 41 & Male :354 & 1st Qu.:23.11 & 1st Qu.: 0.00 & 1st Qu.:4.000 \\
\hline Etrabloz & 34 & & Median :24.38 & Median : 5.00 & Median :4.000 \\
\hline Yvonnand & 34 & & Mean :24.76 & Mean : 6.72 & Mean :4.392 \\
\hline Champmartin & & & 3rd Qu.:26.25 & 3rd Qu.:11.00 & 3rd Qu.:5.000 \\
\hline Lucens & 29 & & Max. :29.25 & Max. :32.00 & Max. :7.000 \\
\hline
\end{tabular}
    (Other) :379
> str(owl)
'data.frame': 599 obs. of 5 variables:
    $ nest : Factor w/ 27 levels "AutavauxTV","Bochet",..: 1 1 1 1 1 1 1 1 1 1 ...
    $ sex : Factor w/ 2 levels "Female","Male": 2 2 2 2 2 2 2 1 2 1...
    $ food : num 22.2 22.4 22.5 22.6 22.6 ...
    $ begging: int 4 0 2 2 2 2 184 18 0 ...
    $ brood : int 5 5 5 5 5 5 5 5 5 5 ...
```

$>$ names(owl)
[1] "nest" "sex" "food" "begging" "brood"
> head(owl)
nest sex food begging brood
1 AutavauxTV Male 22.25
2 AutavauxTV Male 22.3805
3 AutavauxTV Male $22.53 \quad 2 \quad 5$
4 AutavauxTV Male 22.562
5 AutavauxTV Male $22.61 \quad 2 \quad 5$
6 AutavauxTV Male 22.652
\#\#\#\#\# // Create the 'sex10' column //
> owl\$sex10 = ifelse(owl\$sex == 'Male', 0, 1)
> head(owl)
nest sex food begging brood sex10
1 AutavauxTV Male $22.25 \quad 4 \quad 5 \quad 0$
2 AutavauxTV Male 22.38 0 5 0
3 AutavauxTV Male $22.53 \quad 2 \quad 50$
4 AutavauxTV Male 22.5620
5 AutavauxTV Male 22.61 20
6 AutavauxTV Male 22.65020

```
##### // Change the 'sex10' column values to factors //
> owl[, 'sex10'] = as.factor(owl[, 'sex10'])
> str(owl$sex10)
    Factor w/ 2 levels "0","1": 1 1 1 1 1 1 1 2 1 2 ...
```

\#\#\#\#\# // Subset to only include data from broods with 5 chicks //
> owl.5chicks = owl[owl\$brood == 5,]

```
##### // add 'food.category' and 'begging.3' columns //
> owl$food.category = ifelse(owl$food < 25, 'low', 'high')
> head(owl)
        nest sex food begging brood sex10 food.category
    1 AutavauxTV Male 22.25 4 5 0 low
    AutavauxTV Male 22.38 0 5 0 low
3 AutavauxTV Male 22.53 2 5 0 low
4 AutavauxTV Male 22.56 2 5 0 low
5 AutavauxTV Male 22.61 2 5 0 low
6 AutavauxTV Male 22.65 2 5 0 low
> owl$begging. }3\mathrm{ = ifelse(
                                    (owl$begging < 10),
print ('1'),
ifelse(
                (owl$begging > 9 & owl$begging < 20),
                print ('2'),
                ifelse(
                    (owl$begging > 19),
                    print ('3'),
                "NA"
            )
    )
        )
```

[1] "1"
[1] "2"
[1] "3"
> head(owl)
nest sex food begging brood sex10 food.category begging. 3
1 AutavauxTV Male 22.25 4 $5 \quad 0 \quad 0 \quad 1$
2 AutavauxTV Male 22.38 0 0 low 0 low
3 AutavauxTV Male 22.53 2 5 5 0 low 1
4 AutavauxTV Male 22.56 2 5 0 low 1
5 AutavauxTV Male 22.61 2 5 low $0 \quad 1$
6 AutavauxTV Male 22.65 2 5 low $0 \quad 1$
\#\#\#\#\# // Dave the data //
> write.csv(owl, file = "owl_2.csv")

### 10.23.3 Exercise: Data-frames 4

1. Create a new folder that contains the individual.csv data file.
2. Set your working directory to the new folder.
3. Import the individual.csv data and call the data frame ind.
4. Explore the data frame and check structure. [e.g. by using summary(), str(), names(), head()].
5. Create a new column (id) where the individual names from the first column are replaced by numbers starting at one and increasing sequentially. Note that individuals are repeated throughout the dataset so you'll have to make sure each number always matches the same individual.

- HINT: str() \& as.numeric().

6. Save the new data.frame as a .csv file called ind 2.csv.
```
##### // Inport the individual.csv file //
> setwd('~/datasets/individual')
> ind = read.csv('individual.csv')
```

\#\#\#\#\# // Explore the data frame and check structure.

| > summary(ind) |  |  |  |
| :---: | :---: | :---: | :---: |
| individual trait |  |  |  |
| g | :9 | Min. | :2.000 |
| n | :6 | 1st Q | : 4.000 |
| ee | :5 | Media | :6.000 |
| h | :5 | Mean | :5.513 |
| a | :3 | 3rd Qu | :7.000 |
| q | :3 | Max. | :9.000 |

> str(ind)
'data.frame': 39 obs. of 2 variables:
\$ individual: Factor w/ 10 levels "a","ee","g","h",..: 1133333334 ...
\$ trait : int 3426546754 ...
> names(ind)
[1] "individual" "trait"

| head(ind) <br> individual |  |  |
| :--- | ---: | ---: |
|  | trait |  |

\#\#\#\#\# // Create column 'id' //
ind\$id = as.numeric(ind[,1])
$>$ ind
individual trait id
$1 \quad$ a $\quad 31$
$2 \quad a \quad 4 \quad 1$
$3 \quad 9 \quad 2 \quad 3$
$-963$
$9 \quad 43$
$\begin{array}{lll}9 & 6 & 3\end{array}$
g $\quad 7 \quad 3$
$\begin{array}{lll}\mathrm{g} & 5 & 3\end{array}$
4

- 6
71
26
66
76
26
96
ee $8 \quad 2$
ee 6
ee $7 \quad 2$
ee 82
S $\quad 9 \quad 8$
h $\quad 6 \quad 4$

| 25 | g | 3 | 3 |
| :--- | ---: | ---: | ---: |
| 26 | g | 6 | 3 |
| 27 | ee | 7 | 2 |
| 28 | h | 8 | 4 |
| 29 | h | 6 | 4 |
| 30 | q | 5 | 7 |
| 31 | q | 4 | 7 |
| 32 | q | 3 | 7 |
| 33 | t | 6 | 9 |
| 34 | t | 7 | 9 |
| 35 | y | 8 | 10 |
| 36 | y | 8 | 10 |
| 37 | t | 5 | 9 |
| 38 | m | 4 | 5 |
| 39 | m | 3 | 5 |

\#\#\#\#\# //Save the new as 'ind_2.csv' //
> write.csv(ind, file = "ind_2.csv")

### 10.23.4 Exercise: Data-frames 5

1. Create a new folder that contains the RIKZ.csv data file.
2. Set your working directory to the new folder.
3. Import the data (rikz), explore and check structure.
4. reorder the rows of the entire data.frame by increasing values of NAP

- HINT: rikz[order()].

5. create a data.frame called rikz2 that only contains data from beaches one to four for the first week of data collection.
6. what is the mean chalk value for Beach 5 and the mean grain size for beach eight?.
7. save the new data.frame as .csv file called rikz_2.csv.
```
##### // Inport the RIKZ.csv dataset file //
> setwd('~/datasets/RIKZ')
> rikz = read.csv('RIKZ.csv')
##### // Import the data (rikz), explore and check structure //
> summary(rikz)
\begin{tabular}{|c|c|c|c|c|}
\hline Sample & Richness & Week & angle1 & angle2 \\
\hline Min. : 1 & Min. : 0.000 & Min. :1.000 & Min. : 6.00 & Min. 21.00 \\
\hline 1st Qu.:12 & 1st Qu.: 3.000 & 1st Qu.:2.000 & 1st Qu.: 22.00 & 1st Qu.:32.00 \\
\hline Median :23 & Median : 4.000 & Median :2.000 & Median : 32.00 & Median :42.00 \\
\hline Mean :23 & Mean : 5.689 & Mean :2.333 & Mean : 50.31 & Mean :57.78 \\
\hline 3rd Qu.:34 & 3rd Qu.: 8.000 & 3rd Qu.:3.000 & 3rd Qu.: 55.00 & 3rd Qu.:89.00 \\
\hline Max. :45 exposure & \[
\begin{array}{lr}
\text { Max. } & : 22.000 \\
\text { salinity }
\end{array}
\] & Max. :4.000 temperature & \[
\begin{gathered}
\text { Max. } \quad: 312.00 \\
\text { NAP }
\end{gathered}
\] & Max. :96.00 \\
\hline Min. : 8.00 & Min. :26.4 & Min. :15.80 & Min. :-1.3360 & \\
\hline 1st Qu.:10.00 & 1st Qu.:27.1 & 1st Qu.:17.50 & 1st Qu.:-0.3750 & \\
\hline Median :10.00 & Median :27.9 & Median :18.77 & Median : 0.1670 & \\
\hline Mean :10.22 & Mean :28.1 & Mean :18.77 & Mean : 0.3477 & \\
\hline 3rd Qu.:11.00 & 3rd Qu.:29.4 & 3rd Qu.:20.00 & 3rd Qu.: 1.1170 & \\
\hline \[
\begin{array}{lr}
\text { Max. } \quad: 11.00 \\
\text { penetrability }
\end{array}
\] & \begin{tabular}{l}
\[
\text { Max. }: 29.9
\] \\
grainsize
\end{tabular} & Max. :20.80 humus & Max. : 2.2550 chalk & \\
\hline Min. :151.8 & Min. :186.0 & Min. :0.00000 & 0 Min. : 0.85 & \\
\hline 1st Qu.:237.1 & 1st Qu.:222.5 & 1st Qu.:0.00000 & 0 1st Qu.: 2.20 & \\
\hline Median :256.1 & Median :266.0 & Median :0.05000 & 0 Median : 4.75 & \\
\hline Mean :289.4 & Mean :272.5 & Mean :0.05028 & 8 Mean : 7.96 & \\
\hline
\end{tabular}
```

```
3rd Qu.:272.9 3rd Qu.:316.5 3rd Qu.:0.10000 3rd Qu.: 9.150
Max. :624.0 Max. :405.5 Max. :0.30000 Max. :45.750
    sorting1 Beach
Min. : 53.08 Min. :1
1st Qu.: 72.75 1st Qu.:3
Median : 89.03 Median :5
Mean : 97.82 Mean :5
3rd Qu.:115.44 3rd Qu.:7
Max. :248.60 Max. :9
> str(rikz)
'data.frame': 45 obs. of 15 variables:
    $ Sample : int 1 2 3 4 5 6 7 8 9 10 ...
    $ Richness : int 11 10 13 11 10 8 9 8 19 17 ...
    $ Week : int 111111111111 ...
    $ angle1 : int 32 62 65 55 23 129 126 52 26 143 ...
    $ angle2 : int 96 96 96 96 96 89 89 89 89 89 ...
    $ exposure : int 10 10 10 10 10 8 8 8 8 8 ...
    $ salinity : num 29.4 29.4 29.4 29.4 29.4 29.6 29.6 29.6 29.6 29.6 ...
    $ temperature: num 17.5 17.5 17.5 17.5 17.5 20.8 20.8 20.8 20.8 20.8 ...
    $ NAP : num 0.045 -1.036 -1.336 0.616 -0.684 ...
    $ penetrability: num 254 227 237 249 252 ...
    $ grainsize : num 222 200 194 221 202 ...
    $ humus : num 0.05 0.3 0.1 0.15 0.05 0.1 0.1 0.1 0.15 0...
    $ chalk : num 2.05 2.5 3.45 1.6 2.45 2.5 1.85 1.7 2.3 2.6 ...
    $ sorting1 : num 69.8 59 59.2 67.8 57.8 ...
    $ Beach : int 1111112 2 2 2 ...
```


\#\#\#\#\# // Reorder the rows of the entire data.frame by increasing values of NAP. HINT: rikz[order()] //

| > head(rikz) |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Sample | Richness | Week | angle1 | angle2 | exposure | salinity | temperature | NAP |
| 3 | 3 | 13 | 1 | 65 | 96 | 10 | 29.4 | 17.5 | -1.336 |
| 10 | 10 | 17 | 1 | 143 | 89 | 8 | 29.6 | 20.8 | -1.334 |
| 2 | 2 | 10 | 1 | 62 | 96 | 10 | 29.4 | 17.5 | -1.036 |
| 38 | 38 | 7 | 3 | 55 | 32 | 10 | 26.4 | 20.0 | -1.005 |
| 11 | 11 | 6 | 2 | 41 | 42 | 11 | 27.9 | 15.8 | -0.976 |
| 29 | 29 | 6 | 3 | 48 | 36 | 11 | 27.1 | 17.4 | -0.893 |

penetrability grainsize humus chalk sorting1 Beach

| 3 | 237.1 | 194.5 | 0.10 | 3.45 | 59.220 | 1 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |


| 2 | 226.9 | 200.0 | 0.30 | 2.50 | 59.000 | 1 |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: |
| 38 | 382.8 | 355.0 | 0.00 | 16.70 | 133.085 | 8 |
| 11 | 268.4 | 330.5 | 0.05 | 3.40 | 101.330 | 3 |
| 29 | 179.3 | 336.0 | 0.05 | 15.50 | 151.665 | 6 |

\#\#\#\#\# // Create 'rikz2' that only contains Beaches 1-4 from week 1 //
> names(rikz)

| [1] | "Sample" | "Richness" | "Week" |
| ---: | :--- | :--- | :--- | "angle1"

> rikz2 $=$ rikz[rikz\$Week == 1,][rikz[rikz\$Week == 1,]\$Beach > 0
\& rikz[rikz\$Week == 1,]\$Beach < 5,]
> head(rikz2)
Sample Richness Week angle1 angle2 exposure salinity temperature NAP

\#\#\#\#\# // The mean chalk value for Beach 5 //
> rikz[rikz\$Beach == 5,][rikz[rikz\$Beach == 5,]\$chalk,] Sample Richness Week angle1 angle2 exposure salinity temperature NAP

| 22 | 22 | 22 | 4 | 22 | 21 | 10 | 29.9 | 19.8 | -0.503 |
| :--- | ---: | ---: | ---: | ---: | :--- | :--- | :--- | :--- | :--- |
| 25 | 25 | 6 | 4 | 18 | 21 | 10 | 29.9 | 19.8 | 0.054 |
| 25.1 | 25 | 6 | 4 | 18 | 21 | 10 | 29.9 | 19.8 | 0.054 |

penetrability grainsize humus chalk sorting1 Beach

| 22 | 256.1 | 265.0 | 0.0 | 1.6 | 89.035 | 5 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 25 | 231.1 | 254.5 | 0.1 | 2.1 | 86.170 | 5 |
| 25.1 | 231.1 | 254.5 | 0.1 | 2.1 | 86.170 | 5 |

\#\#\#\#\# // The mean grain size for Beach 8 //
> mean(rikz[rikz\$Beach == 8,][,'grainsize'])
[1] 327.9
\#\#\#\#\# // Save the new data.frame 'rikz_2.csv' //

```
> write.csv(rikz, file = "rikz_2.csv")
```


## 11. Linear Models, predictions and probability distributions

### 11.1 Some terms

- Mean - is a set of values is the quantity commonly called the mean or the average. To get the mean sum the number of of values and divide the result by the number of values.

$\operatorname{Im}(\mathbf{y} \sim 1)$ will return the mean value.
- Regression - A method for fitting a curve or a straight line through a set of points using some goodness-of-fit criterion. The most common type of regression is linear regression.
- Linear Regression - A regression that is linear in the unknown parameters used in the fit. It is defined by the intercept point (a) of the best fit line where the $x$ axis is equal to zero plus the slope ( $\boldsymbol{\beta}$ ) of the best fit line times the value of $x$. The intercept point ( $a$ ) is the mean.



## Illustration 6: Linear regression model

- Multiple Regression - A regression giving conditional expectation values of a given variable in terms of two or more other variables.
- Null Hypothesis $\left(\mathrm{H}_{0}\right)$ - A $\mathrm{H}_{0}$ is a statistical hypothesis that is tested for possible rejection under the assumption that it is true (usually that observations are the result of chance). It means that there is no expectation of change.
- Alternative Hypothesis $\left(\mathrm{H}_{1}\right)$ - is the hypothesis used in hypothesis testing that is contrary to the $\mathrm{H}_{0}$. It is usually taken to be that the observations are the result of a real effect (with some amount of chance variation superposed). In the p-test values less than 0.05 indicate a significance and indicate contrary to the $\mathrm{H}_{0}$.
- Estimate - Mean expectation of the mean, the Null Hypothesis $\left(\mathrm{H}_{0}\right)$.
- Standard Error - This gives one standard deviation (ठ) of the certainty of the accuracy of the Estimate (mean). $2 \delta$ gives the certainty of accuracy for $95 \%$ of values. mean $=6.7195$ and $2 \delta=0.55$, therefore there is a $95 \%$ chance of values giving a mean in the range of $6.17-7.27$.
- t-value - The coefficient divided by its standard error. The standard error is an estimate of the standard deviation of the coefficient, the amount it varies across cases. It can be thought of as a measure of the precision with which the regression coefficient is measured. This value says how many standard deviations from zero. For example:
the Estimate / Std. Error $=t$-value $==>6.7195 / 0.2726=24.64967$
- p-value - Shows the expectation that the Estimate is accurate for values. A small $p$-value (typically $\leq 0.05$ ) indicates strong evidence against the $\mathrm{H}_{0}$, so you reject the $\mathrm{H}_{0}$. A large p -value ( $>0.05$ ) indicates weak evidence against the $\mathrm{H}_{0}$, so you shouldn't reject $\mathrm{H}_{0}$. The p -value of 0.05 means that there is less than a $5 \%$ chance of the results arising due to random chance.
- t-test - Refers to any statistical hypothesis test in which the test statistic follows a t-distribution under the $\mathrm{H}_{0}$. A t-test is most commonly applied when the test statistic would follow a normal distribution if the value of a scaling term in the test statistic were known. Here is an example with the built-in mtcars dataset. It shows a t-test to compare vehicles in rows 1 to 10 with vehicles in rows 11 to 20 for column 1 which is miles per gallon ( mpg ). The $\mathrm{H}_{0}$ is that there there is little to no statistical difference between the groups in terms of mpg. As the p-value from the test is 0.8744 , it concurs with the $\mathrm{H}_{0}$ and sets the expectation that there is indeed little statistical expectation of a significant difference.

```
> data(mtcars)
> head(mtcars[1])
Mazda RX4 mpg
Mazda RX4 Wag 21.0
Datsun 710 22.8
Hornet 4 Drive 21.4
Hornet Sportabout 18.7
Valiant 18.1
> a = t.test(mtcars[1:10,1],mtcars[11:20,1])
> print (a)
    Welch Two Sample t-test
data: mtcars[1:10, 1] and mtcars[11:20, 1]
t = 0.16185, df = 10.892, p-value = 0.8744
alternative hypothesis: true difference in means is not equal to 0
95 percent confidence interval:
    -6.055477 7.015477
sample estimates:
mean of }x\mathrm{ mean of }
        20.37 19.89
> str(a)
List of 9
$ statistic : Named num 0.162
        ..- attr(*, "names")= chr "t"
$ parameter : Named num 10.9
    ..- attr(*, "names")= chr "df"
$ p.value : num 0.874
$ conf.int : atomic [1:2] -6.06 7.02
        ..- attr(*, "conf.level")= num 0.95
$ estimate : Named num [1:2] 20.4 19.9
        ..- attr(*, "names")= chr [1:2] "mean of x" "mean of y"
$ null.value : Named num 0
        ..- attr(*, "names")= chr "difference in means"
$ alternative: chr "two.sided"
$ method : chr "Welch Two Sample t-test"
$ data.name : chr "mtcars[1:10, 1] and mtcars[11:20, 1]"
- attr(*, "class")= chr "htest"
```

```
> unclass(a)
    $statistic
            t
    0.1618478
$parameter
            df
10.89198
$p.value
[1] 0.8743889
$conf.int
[1] -6.055477 7.015477
attr(,"conf.level")
[1] 0.95
$estimate
mean of x mean of }
    20.37 19.89
$null.value
difference in means
\$alternative
[1] "two.sided"
\$method
[1] "Welch Two Sample t-test"
\$data. name
[1] "mtcars[1:10, 1] and mtcars[11:20, 1]"
\# Extract the mean of \(y\)
> a\$estimate[2]
mean of \(y\)
19.89
\# Extract the 'p' value
> a\$p.value
[1] 0.8743889
```

- Dependent Variable (DV) - The variable being tested and measured in an experiment.
- Independent Variable (IV) - The variable that is changed or controlled in a scientific experiment to test the effects on the DV.
- Analysis of Variance (ANOVA) - is a collection of statistical models and their associated estimation procedures used to analyse the differences among group means in a sample.

$$
y=\alpha+\beta x_{1}+\beta x_{2}
$$

- Multivarite ANOVA (MANOVA) - is a procedure for comparing multivariate sample means. As a multivariate procedure, it is used when there are two or more dependent variables, and is typically followed by significance tests involving individual dependent variables separately.

Analysis of covariance (ANCOVA) - is a general linear model which blends ANOVA and regression. ANCOVA evaluates whether the means of a DV are equal across levels of a categorical IV.

- Generalised Linear Model (GLM) is a flexible generalisation of ordinary linear regression that allows for response variables that have error distribution models other than a normal distribution.
- Generalised Linear Mixed Model (GLMM) - is an extension to GLM in which the linear predictor contains random effects in addition to the usual fixed effects. They also inherit from GLMs the idea of extending linear mixed models to nonnormal data.


### 11.2 Demonstration

Working with the owl_data.csv file once more to demonstrate linear models, predictions and probability distributions.

```
> setwd('~/datasets/owl_data_2')
> owl = read.csv('owl_data.csv')
##### // Inspect the data //
> str(owl)
'data.frame': 599 obs. of 5 variables:
    $ nest : Factor w/ 27 levels "AutavauxTV","Bochet",..: 1 1 1 1 1 1 1 1 1 1 ...
    $ sex : Factor w/ 2 levels "Female","Male": 2 2 2 2 2 2 2 1 2 1...
    $ food : num 22.2 22.4 22.5 22.6 22.6 ...
    $ begging: int 4 0 2 2 2 2 184 18 0 ...
$ brood : int 5 5 5 5 5 5 5 5 5 5 ...
\begin{tabular}{|c|c|c|c|c|}
\hline \multicolumn{5}{|l|}{> summary(owl)} \\
\hline nest & sex & food & begging & brood \\
\hline Oleyes : 52 & Female:245 & Min. :21.71 & Min. : 0.00 & Min. :1.000 \\
\hline Moutet : 41 & Male :354 & 1st Qu.:23.11 & 1st Qu.: 0.00 & 1st Qu.:4.000 \\
\hline Etrabloz : 34 & & Median :24.38 & Median : 5.00 & Median :4.000 \\
\hline Yvonnand : 34 & & Mean :24.76 & Mean : 6.72 & Mean :4.392 \\
\hline Champmartin: 30 & & 3rd Qu.:26.25 & 3rd Qu.:11.00 & 3rd Qu.:5.000 \\
\hline Lucens : 29 & & Max. \(: 29.25\) & Max. :32.00 & Max. :7.000 \\
\hline
\end{tabular}
> head(owl)
        nest sex food begging brood
1 AutavauxTV Male 22.25 4 5
2 AutavauxTV Male 22.38 0 5
3 AutavauxTV Male 22.53 2 5
4 AutavauxTV Male 22.56 2 5
5 AutavauxTV Male 22.61 2 5
6 AutavauxTV Male 22.65 2 5
```

The purpose of this is to explain the begging rate of the chicks in the nest.
Response variable $=\mathrm{y}=$ begging
$\operatorname{Im}()$ : is used to fit linear models. It can be used to carry out regression, single stratum analysis of variance and analysis of covariance.

### 11.3 The mean()

Question 1: What is the begging rate of the sampled population?

```
mean(owl$begging)
[1] 6.719533
> lm(begging ~ 1, data = owl)
Call:
lm(formula = begging ~ 1, data = owl)
Coefficients:
(Intercept)
    6.72
```

$\operatorname{Im}(\mathbf{y} \sim \mathbf{1})$ returns the mean value $(\mathbf{a})$, the intercept where $\beta \boldsymbol{x}=\mathbf{1}$.


Now the statistics are given in the summary() of the data. Estimate, Standard error, $t$ value and $p$-value.

```
> mod = lm(begging ~ 1, data = owl)
```

```
> summary(mod)
Call:
lm(formula = begging ~ 1, data = owl)
Residuals:
    Min 1Q Median 3Q Max
    -6.72 -6.72 -1.72 4.28 25.28
Coefficients:
    Estimate Std. Error t value Pr(>|t|)
(Intercept) 6.7195 0.2726 24.65 <2e-16 ***
Signif. codes: 0 '***' 0.001 ،**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
Residual standard error: 6.671 on 598 degrees of freedom
```


### 11.4 The t-test

Question 2: What is the begging rate of males versus females?
In this case $R$ coded females $=0$ and males $=1$. So therefore females indicate the intercept ( ${ }^{*} 0$ ) and males the difference from the intercept. This is indicated by sexMale in the second line of the output, males have been labelled internally as $=1$, therefore sexFemale must be $=0$. So the intercept of females 6.0122 plus the difference 1.1968 gives the intercept value for males $=7.209$.
In $R$ the linear modelling is carried out using the $\operatorname{Im}()$ function. The DV is placed to the left side of the tilde and the IV or IVs are placed to the right. As such the DV are the $y$ axis and the IVs are on the $x$ axis of associated graphs.

## $\operatorname{Im}(D V$ ~ IV(s), data=<dataset>)

```
> lm(begging ~ sex, data = owl)
Call:
lm(formula = begging ~ sex, data = owl)
Coefficients:
(Intercept) sexMale
    6.012 1.197
> mod2 = lm(begging ~ sex, data = owl)
```

```
> summary(mod2)
Call:
lm(formula = begging ~ sex, data = owl)
Residuals:
    Min 1Q Median 3Q Max
-7.209 -6.012 -1.209 4.791 24.791
Coefficients:
    Estimate Std. Error t value Pr(>|t|)
l(Intercept) 6.0122 
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Residual standard error: 6.65 on 597 degrees of freedom Multiple R-squared: 0.007793, Adjusted R-squared: 0.006131 F-statistic: 4.689 on 1 and 597 DF, p-value: 0.03075

$$
\begin{gathered}
\operatorname{Im}(y \sim x 1+x 2+\ldots) \\
y=a+\beta x_{1}+\beta x_{2}+\beta x_{n}
\end{gathered}
$$



Illustration 8: Mean begging rate based on sex

### 11.5 ANOVA, the Analysis of Variance

Comparing the means of greater than two groups.
Question 3: How does the begging rate change as nest size increases?
Important: First change owl\$brood to a factor.

```
> str(owl$brood)
    int [1:599] 5 5 5 5 5 5 5 5 5 5 ...
> owl$brood = as.factor(owl$brood)
> str(owl$brood)
    Factor w/ 7 levels "1","2","3","4",..: 5 5 5 5 5 5 5 5 5 5 ...
```

This is an ANOVA. the Im(begging ~ brood, data $=$ owl) doesn't give much information but the summary (Im(begging ~ brood, data $=$ owl)) gives more useful information with brood1 indicating the intercept and all other broods are indicated as the difference from brood1.

Can't be certain that the birds didn't beg at a negative rate. Obviously this isn't possible.
$t$-value and $p$-values demonstrate the certainty of difference between each brood and brood1. However there is a high level of uncertainty of $\mathrm{H}_{0}$.

For example brood 7 p -value is less than 0.05 and therefore indicates that it doesn't have the same begging rate as brood1, i.e. reject the $\mathrm{H}_{0}$.

This test is only comparing each brood relative to brood1. To review this further posthoc tests which will carry out statistics between each brood group.

```
> lm(begging ~ brood, data = owl)
Call:
lm(formula = begging ~ brood, data = owl)
Coefficients
(Intercept) brood2 brood3 brood4 brood5 brood6
\begin{tabular}{lllll}
4.0000 & -0.5263 & 0.4237 & 3.0905 & 2.6404
\end{tabular}
    brood7
    7.5000
> mod3 = lm(begging ~ brood, data = owl)
```

```
> summary(mod3)
Call:
lm(formula = begging ~ brood, data = owl)
Residuals:
\begin{tabular}{rrrrr} 
Min & \(1 Q\) & Median & \(3 Q\) & Max \\
-11.500 & -5.090 & -1.500 & 4.743 & 25.360
\end{tabular}
Coefficients:
\begin{tabular}{lrrrr} 
& Estimate & Std. Error & t value & \(\operatorname{Pr}(>|t|)\) \\
(Intercept) & 4.0000 & 3.2477 & 1.232 & 0.2186 \\
brood2 & -0.5263 & 3.4144 & -0.154 & 0.8775 \\
brood3 & 0.4237 & 3.3560 & 0.126 & 0.8996 \\
brood4 & 3.0905 & 3.2785 & 0.943 & 0.3462 \\
brood5 & 2.6404 & 3.2761 & 0.806 & 0.4206 \\
brood6 & 4.5385 & 3.4886 & 1.301 & 0.1938 \\
brood7 & 7.5000 & 3.4334 & 2.184 & 0.0293 *
\end{tabular}
Signif. codes: 0 ،***' 0.001 ،**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
Residual standard error: 6.495 on 592 degrees of freedom Multiple R-squared: 0.06141, Adjusted R-squared: 0.0519
F-statistic: 6.456 on 6 and 592 DF, p-value: 1.317e-06
```

$\operatorname{lm}(y \sim x 1+x 2+\ldots)$
$y=a+\beta x_{1}+\beta x_{2}+\beta x_{n}$

anova() - Compute analysis of variance (or deviance) tables for one or more fitted model objects.
$\operatorname{aov}()$ - Fit an analysis of variance model by a call to Im for each stratum.
tukey() - Create a set of confidence intervals on the differences between the means of the levels of a factor with the specified family-wise probability of coverage. The intervals are based on the Studentised range statistic, Tukey Honest Significant Difference method.

```
> post = aov(mod3)
> TukeyHSD(post)
    Tukey multiple comparisons of means
        95% family-wise confidence level
Fit: aov(formula = mod3)
```

\$brood
diff lwr upr p adj
2-1 -0.5263158-10.6277301 9.575099 0.9999989
$3-1 \quad 0.4237288-9.504999010 .3524570 .9999997$
$4-1 \quad 3.0904762-6.608961912 .7899140 .9654242$
$5-1 \quad 2.6403509-7.051928112 .332630 \quad 0.9843572$
$6-1 \quad 4.5384615 \quad-5.782574414 .859498 \quad 0.8514925$
$7-1 \quad 7.5000000 \quad-2.657847317 .657847 \quad 0.3055212$
$\begin{array}{lllll}3-2 & 0.9500446 & -3.0470773 & 4.947167 & 0.9923990\end{array}$
$\begin{array}{lllll}4-2 & 3.6167920 & 0.2291015 & 7.004482 & 0.0276208\end{array}$
$5-2 \quad 3.1666667-0.2004714 \quad 6.533805 \quad 0.0809328$
$\begin{array}{llll}6-2 & 5.0647773 & 0.1738541 & 9.955701\end{array} 0.0368230$
$7-2 \quad 8.0263158 \quad 3.489884612 .5627470 .0000048$
$\begin{array}{lllll}4-3 & 2.6667474 & -0.1647738 & 5.498269 & 0.0800918\end{array}$
$\begin{array}{lllll}5-3 & 2.2166221 & -0.5902774 & 5.023521 & 0.2284480\end{array}$
$\begin{array}{lllll}6-3 & 4.1147327 & -0.4087831 & 8.638249 & 0.1022524\end{array}$
7 -3 $7.0762712 \quad 2.938606011 .2139360 .0000116$
5-4 -0.4501253 -2.2880991 1.387848 0.9910760
$\begin{array}{lllll}6-4 & 1.4479853 & -2.5472191 & 5.443190 & 0.9360239\end{array}$
$\begin{array}{lllll}7-4 & 4.4095238 & 0.8570970 & 7.961951 & 0.0048667\end{array}$
$\begin{array}{lllll}6-5 & 1.8981107 & -2.0796816 & 5.875903 & 0.7955024\end{array}$
$\begin{array}{lllll}7-5 & 4.8596491 & 1.3268162 & 8.392482 & 0.0010456\end{array}$

| $7-6$ | 2.9615385 | -2.0448994 | 7.967976 | 0.5824679 |
| :--- | :--- | :--- | :--- | :--- |

### 11.6 Regression

Compare the means of a continuous variable.
Question 4: How does begging rate change as food increases?
This demonstrates that the expectation line intercepts the $y$ axis at a 26.9 begging rate. This may or may not have an actual biological meaning as the samples taken may not actually have gone down to zero food. The same can be said of the line crossing the $x$ axis. However it is useful mathematically. The slope of negative ( -0.81 ) means the expectation is that begging reduces by 0.81 for every 1 unit of food added. The very low $p$-value means that there is a very high confidence that more food decreases the begging rate.

```
> lm(begging ~ food, data = owl)
Call:
lm(formula = begging ~ food, data = owl)
Coefficients:
(Intercept) food
> mod4 = lm(begging ~ food, data = owl)
```


## > summary(mod4)

Call:
$\operatorname{lm}(f o r m u l a=$ begging $\sim$ food, data $=$ owl)
Residuals:
Min 1Q Median 3Q Max
$-9.155-5.253-1.298 \quad 4.45425 .544$

Coefficients:
Estimate Std. Error $t$ value $\operatorname{Pr}(>|t|)$
(Intercept) $26.9738 \quad 3.4431 \quad 7.8342 .17 \mathrm{e}-14$ ***
food -0.8181 0.1387-5.900 6.09e-09 ***

Signif. codes: 0 "***' 0.001 '**' 0.01 '*' $0.05{ }^{\prime} .{ }^{\prime} 0.1$ ' ' 1
Residual standard error: 6.49 on 597 degrees of freedom Multiple R-squared: 0.0551, Adjusted R-squared: 0.05351
F-statistic: 34.81 on 1 and 597 DF, p-value: 6.092e-09

$$
\begin{gathered}
\operatorname{lm}(\mathrm{y} \sim \mathrm{x} 1+\mathrm{x} 2+\ldots) \\
\boldsymbol{y}=\boldsymbol{a}+\beta x_{1}+\beta x_{2}+\beta x_{n}
\end{gathered}
$$



Illustration 10: Regression

### 11.7 Multiple regression

Compare the means of a continuous variable plus a group variable.
Question 5a: How does begging rate change as food and sex increase?

```
> lm(begging ~ food + sex, data = owl)
Call:
lm(formula = begging ~ food + sex, data = owl)
Coefficients:
(Intercept) food sexMale
    26.4475 -0.8276 1.2892
mod5a = lm(begging ~ food + sex, data = owl)
> summary(mod5a)
```

Call:
$\operatorname{lm}(f o r m u l a=$ begging $\sim$ food + sex, data $=$ owl)
Residuals:
Min 1Q Median 3Q Max
-9.711 -5.244 -1.597 4.700 25.020
Coefficients:
Estimate Std. Error $t$ value $\operatorname{Pr}(>|\mathrm{t}|)$

| (Intercept) | 26.4475 | 3.4365 | 7.696 | $5.85 \mathrm{e}-14$ | *** |
| :--- | ---: | ---: | ---: | ---: | ---: |
| food | -0.8276 | 0.1382 | -5.990 | $3.63 \mathrm{e}-09$ *** |  |
| sexMale | 1.2892 | 0.5374 | 2.399 | 0.0168 * |  |



Residual standard error: 6.464 on 596 degrees of freedom Multiple R-squared: 0.06413, Adjusted R-squared: 0.06099 F-statistic: 20.42 on 2 and 596 DF, p-value: $2.642 e-09$

$$
\begin{gathered}
\operatorname{lm}(y \sim x 1+x 2+\ldots) \\
y=a+\beta x_{1}+\beta x_{2}+\beta x_{n}
\end{gathered}
$$



Question 5b: How does begging rate change as food and brood increase?
The data is a multidimensional model with both food and brood. In this case brood is now continuous data showing brood size (as.numeric(owl\$brood)).

```
> owl$brood = as.numeric(owl$brood)
> lm(begging ~ food + brood, data = owl)
Call:
lm(formula = begging ~ food + brood, data = owl)
Coefficients:
(Intercept) food brood
    21.0447 -0.7877 1.1786
> mod5b = lm(begging ~ food + brood, data = owl)
> summary(mod5b)
Call:
lm(formula = begging ~ food + brood, data = owl)
Residuals:
\begin{tabular}{rrrrr} 
Min & \(1 Q\) & Median & \(3 Q\) & Max \\
-10.981 & -5.250 & -1.300 & 4.514 & 24.818
\end{tabular}
Coefficients:
    Estimate Std. Error t value Pr(>|t|)
(Intercept) 21.0447 3.5561 5.918 5.50e-09 ***
food -0.7877 0.1358-5.799 1.08e-08 ***
brood 1.1786 0.2258 5.220 2.47e-07 ***
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Residual standard error: 6.352 on 596 degrees of freedom Multiple R-squared: 0.09641, Adjusted R-squared: 0.09338 F-statistic: 31.79 on 2 and 596 DF, p-value: 7.583e-14

$$
\begin{gathered}
\operatorname{lm}(\mathrm{y} \sim \mathrm{x} 1+\mathrm{x} 2+\ldots) \\
\boldsymbol{y}=\boldsymbol{a}+\boldsymbol{\beta} \mathrm{x}_{1}+\boldsymbol{\beta} \mathrm{x}_{2}+\boldsymbol{\beta} \mathrm{x}_{\boldsymbol{n}}
\end{gathered}
$$



Illustration 12: Another multiple regression

### 11.8 ANCOVA, the Analysis of Covariance

Comparing the means of multiple variables and allow for interactions between them.
Question 5c: How does the begging rate change as food \& sex increase, while allowing for the food effect of each sex to differ?

The interaction of food \& sex: an ANCOVA. If the effect of both food and sex change relative to each other then the interaction of both must be accounted for. This allows the relationship between males and females to change. As the $p$-value is quite large there is no reason to reject the $H_{0}$. In other words there is little evidence that there is a divergence between the begging rate of males and females given greater levels of food.

```
> lm(begging ~ food + sex + food*sex, data = owl)
Call:
lm(formula = begging ~ food + sex + food * sex, data = owl)
Coefficients:
    (Intercept) food sexMale food:sexMale
        23.6642 -0.7149 6.0699 -0.1933
> mod5c = lm(begging ~ food + sex + food*sex, data = owl)
> summart(mod5c)
Error in summart(mod5c) : could not find function "summart"
```

```
> summary(mod5c)
Call:
lm(formula = begging ~ food + sex + food * sex, data = owl)
Residuals:
    Min 1Q Median 3Q Max
-9.955 -5.241 -1.461 4.515 25.042
Coefficients:
    Estimate Std. Error t value Pr(>|t|)
(Intercept) 23.6642 5.3035 4.462 9.71e-06 ***
food -0.7149 0.2141 -3.338 0.000895 ***
sexMale 6.0699 6.9569 0.873 0.383286
food:sexMale -0.1933 0.2804 -0.689 0.490936
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
Residual standard error: 6.467 on 595 degrees of freedom Multiple R-squared: 0.06488, Adjusted R-squared: 0.06016 F-statistic: 13.76 on 3 and 595 DF, p-value: 1.094e-08
```



Illustration 13: ANCOVA

### 11.9 Linear model summary

| Function | R function | Result |
| :---: | :---: | :---: |
| $y=\alpha+\beta x$ | Im(begging ~ 1) | The mean of $y$ |
| $y=\alpha+\beta x$ | Im(begging ~ sex) | t-test |
| $y=\alpha+\beta x_{1}+\beta x_{2}$ | Im(begging ~ brood size) | ANOVA |
| $y=\alpha+\beta x$ | Im(begging ~ food) | Simple regression |
| $y=\alpha+\beta x_{1}+\beta x_{2}$ | Im(begging ~ food + sex) | Multiple regression |
| $y=\alpha+\beta x_{1}+\beta x_{2}$ | Im(begging ~ food + brood) | Multiple regression |
| $y=\alpha+\beta x_{1}+\beta x_{2}$ | Im(begging $\sim$ food + sex + food*sex) | ANCOVA |

All linear models are about calculating the mean of a particular group or groups. The statistics simply tell you how confident you are that the means differ between groups.

### 11.10 Exercise: Linear models

1. Import the RIKZ.csv data into an object called rikz (inspect the data etc).
2. What is the average species richness across all samples?
3. How does species richness vary with sand grainsize?

- HINT: simple regression.

4. How does species richness vary at the different beaches?

- HINT: ANOVA.

5. How does species richness vary with sampling time (week $1 \& 2$ versus week 3\&4)?

- HINT: t-test (will need to create a 2-category variable from week).

6. How does species richness vary with NAP and beach angle; is there any evidence of an interaction effect between them?

- HINT: multiple regression.

Question 1: Import the 'RIKZ.csv' data into an object called rikz.

```
> setwd('~/datasets/RIKZ_2')
> rikz = read.csv('RIKZ.csv')
```


## Explore the data frame.



| head(rikz) |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Sample | Richness | Week ang | gle1 an | angle2 | exposure | salinity | temperature | NAP |
| 1 | 11 | 1 | 32 | 96 | 10 | 29.4 | 17.5 | 0.045 |
| 2 | 10 | 1 | 62 | 96 | 10 | 29.4 | 17.5 | -1.036 |
| 3 | 13 | 1 | 65 | 96 | 10 | 29.4 | 17.5 | -1.336 |
| 4 | 11 | 1 | 55 | 96 | 10 | 29.4 | 17.5 | 0.616 |
| 5 | 10 | 1 | 23 | 96 | 10 | 29.4 | 17.5 | -0.684 |
| 6 | 8 | 1 | 129 | 89 | 8 | 29.6 | 20.8 | 1.190 |
| penetrability |  | grainsize | humus | chalk | sorting1 | Beach |  |  |
|  | 253.9 | 222.5 | 0.05 | 2.05 | 69.830 | 1 |  |  |
|  | 226.9 | 200.0 | 0.30 | 2.50 | 59.000 | 1 |  |  |
|  | 237.1 | 194.5 | 0.10 | 3.45 | 59.220 | 1 |  |  |
|  | 248.6 | 221.0 | 0.15 | 1.60 | 67.750 | 1 |  |  |
|  | 251.9 | 202.0 | 0.05 | 2.45 | 57.760 | 1 |  |  |
|  | 250.1 | 192.5 | 0.10 | 2.50 | 53.075 | 2 |  |  |

Question 2: What is the average species richness across all samples?

```
Mean : 5.689 # Extracted from the summary(rikz)
# Alternatively extract from the intercept (\alpha) where y = 0
> lm (Richness ~ 1, rikz)
Call:
lm(formula = Richness ~ 1, data = rikz)
Coefficients:
(Intercept)
    5.689
```


## Question 3: How does species richness vary with sand grainsize?

```
> rikz 1 = lm(Richness ~ grainsize, data=rikz)
> summary(rikz1)
Call:
lm(formula = Richness ~ grainsize, data = rikz)
Residuals:
    Min 1Q Median 3Q Max
-8.4833-3.6733-0.2693 2.1872 16.0701
Coefficients:
    Estimate Std. Error t value Pr(>|t|)
(Intercept) 14.49529 3.40326 4.259 0.000109 ***
grainsize -0.03232 0.01222 -2.644 0.011386 *
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
Residual standard error: 4.694 on 43 degrees of freedom
Multiple R-squared: 0.1399, Adjusted R-squared: 0.1198
F-statistic: 6.991 on 1 and 43 DF, p-value: 0.01139
```

Answer, Very little as the slope of the line is almost zero with a $p$-value $<0.05$ means that the model demonstrates a high probability of $\mathrm{H}_{0}$.


Question 4: How does species richness vary at the different beaches?

```
> rikz$Beach = as.factor(rikz$Beach)
> rikz2 = lm(Richness ~ Beach, data=rikz)
> summary(rikz2)
Call:
lm(formula = Richness ~ Beach, data = rikz)
Residuals:
\begin{tabular}{rrrrr} 
Min & \(1 Q\) & Median & \(3 Q\) & Max \\
-7.4 & -1.4 & -0.2 & 1.0 & 14.6
\end{tabular}
Coefficients:
                            Estimate Std. Error t value Pr(>|t|)
(Intercept) 11.000 1.761 6.245 3.27e-07 ***
\begin{tabular}{lllll} 
Beach2 & 1.200 & 2.491 & 0.482 & 0.63289
\end{tabular}
Beach3 -7.600 \(2.491-3.051\) 0.00426 **
Beach4 -8.600 2.491 -3.453 0.00144 **
\begin{tabular}{lllll} 
Beach5 & -3.600 & 2.491 & -1.445 & 0.15703
\end{tabular}
Beach6 -7.000 \(2.491-2.810 \quad 0.00796\) **
Beach7 -8.800 2.491 -3.533 0.00115 **
Beach8 -7.000 \(2.491-2.810 \quad 0.00796\) **
Beach9 -6.400 2.491 -2.569 0.01448 *
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Residual standard error: 3.938 on 36 degrees of freedom Multiple R-squared: 0.4931, Adjusted R-squared: 0.3805 F-statistic: 4.378 on 8 and 36 DF, p-value: 0.0009179

As the p -value is less than 0.05 there is a high level of confidence that at least one of the pairs of beaches vary from each other in species richness. Carrying out fit an analysis of variance model test $(\operatorname{aov}())$ test to investigate further.

```
> rikz3 = aov(rikz2)
> summary (rikz3)
    Df Sum Sq Mean Sq F value Pr(>F)
Beach 8 543.2 67.91 4.378 0.000918 ***
Residuals }\begin{array}{llll}{36}&{558.4}&{15.51}
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

The resulting $p$-value of 0.000918 says that with a high level of probability that there is a variance between beaches. Compute the Tukey Honest Significant Differences between the beaches.

```
> rikz4 = TukeyHSD(rikz3)
> print(rikz4)
    Tukey multiple comparisons of means
        95% family-wise confidence level
Fit: aov(formula = rikz2)
```

\$Beach

|  | diff | lwr | upr | p adj |
| ---: | ---: | ---: | ---: | ---: |
| $2-1$ | $1.200000 \mathrm{e}+00$ | -7.01263 | 9.41262978 | 0.9999026 |
| $3-1$ | $-7.600000 \mathrm{e}+00$ | -15.81263 | 0.61262978 | 0.0882779 |
| $4-1$ | $-8.600000 \mathrm{e}+00$ | -16.81263 | -0.38737022 | 0.0342201 |
| $5-1$ | $-3.600000 \mathrm{e}+00$ | -11.81263 | 4.61262978 | 0.8723088 |
| $6-1$ | $-7.000000 \mathrm{e}+00$ | -15.21263 | 1.21262978 | 0.1475693 |
| $7-1$ | $-8.800000 \mathrm{e}+00$ | -17.01263 | -0.58737022 | 0.0279853 |
| $8-1$ | $-7.000000 \mathrm{e}+00$ | -15.21263 | 1.21262978 | 0.1475693 |
| $9-1$ | $-6.400000 \mathrm{e}+00$ | -14.61263 | 1.81262978 | 0.2347476 |
| $3-2$ | $-8.800000 \mathrm{e}+00$ | -17.01263 | -0.58737022 | 0.0279853 |
| $4-2$ | $-9.800000 \mathrm{e}+00$ | -18.01263 | -1.58737022 | 0.0097691 |
| $5-2$ | $-4.800000 \mathrm{e}+00$ | -13.01263 | 3.41262978 | 0.6005725 |
| $6-2$ | $-8.200000 \mathrm{e}+00$ | -16.41263 | 0.01262978 | 0.0506097 |
| $7-2$ | $-1.000000 \mathrm{e}+01$ | -18.21263 | -1.78737022 | 0.0078509 |
| $8-2$ | $-8.200000 \mathrm{e}+00$ | -16.41263 | 0.01262978 | 0.0506097 |
| $9-2$ | $-7.600000 \mathrm{e}+00$ | -15.81263 | 0.61262978 | 0.0882779 |
| $4-3$ | $-1.000000 \mathrm{e}+00$ | -9.21263 | 7.21262978 | 0.9999757 |
| $5-3$ | $4.000000 \mathrm{e}+00$ | -4.21263 | 12.21262978 | 0.7953899 |
| $6-3$ | $6.000000 \mathrm{e}-01$ | -7.61263 | 8.81262978 | 0.9999995 |
| $7-3$ | $-1.200000 \mathrm{e}+00$ | -9.41263 | 7.01262978 | 0.9999026 |
| $8-3$ | $6.000000 \mathrm{e}-01$ | -7.61263 | 8.81262978 | 0.9999995 |
| $9-3$ | $1.200000 \mathrm{e}+00$ | -7.01263 | 9.41262978 | 0.9999026 |
| $5-4$ | $5.000000 \mathrm{e}+00$ | -3.21263 | 13.21262978 | 0.5484761 |
| $6-4$ | $1.600000 \mathrm{e}+00$ | -6.61263 | 9.81262978 | 0.9991816 |
| $7-4$ | $-2.000000 \mathrm{e}-01$ | -8.41263 | 8.01262978 | 1.0000000 |
| $8-4$ | $1.600000 \mathrm{e}+00$ | -6.61263 | 9.81262978 | 0.9991816 |
| $9-4$ | $2.200000 \mathrm{e}+00$ | -6.01263 | 10.41262978 | 0.9925552 |
| $6-5$ | $-3.400000 \mathrm{e}+00$ | -11.61263 | 4.81262978 | 0.9034383 |
| $7-5$ | $-5.200000 \mathrm{e}+00$ | -13.41263 | 3.01262978 | 0.4968860 |
| $8-5$ | $-3.400000 \mathrm{e}+00$ | -11.61263 | 4.81262978 | 0.9034383 |
| $9-5$ | $-2.800000 \mathrm{e}+00$ | -11.01263 | 5.41262978 | 0.9664876 |
| $7-6$ | $-1.800000 \mathrm{e}+00$ | -10.01263 | 6.41262978 | 0.9981055 |
| $8-6$ | $8.881784 \mathrm{e}-16$ | -8.21263 | 8.21262978 | 1.0000000 |
| $9-6$ | $6.000000 \mathrm{e}-01$ | -7.61263 | 8.81262978 | 0.9999995 |
| $8-7$ | $1.800000 \mathrm{e}+00$ | -6.41263 | 10.01262978 | 0.9981055 |
| $9-7$ | $2.400000 \mathrm{e}+00$ | -5.81263 | 10.61262978 | 0.9869248 |
| $9-8$ | $6.000000 \mathrm{e}-01$ | -7.61263 | 8.81262978 | 0.9999995 |

Looking over the p -values there are a number of beach pairs that how a value lower than 0.05 , in other words there is a difference in species diversity for these beach pairs. Convert the list to a dataframe and pull out that data.

```
> df.rikz4 = data.frame(unclass(rikz4))
> names(df.rikz4)
[1] "Beach.diff" "Beach.lwr" "Beach.upr" "Beach.p.adj"
> df.rikz4[df.rikz4$Beach.p.adj < 0.05,]
    Beach.diff Beach.lwr Beach.upr Beach.p.adj
4-1 -8.6 -16.81263 -0.3873702 0.034220065
7-1 -8.8 -17.01263 -0.5873702 0.027985274
3-2 -8.8 -17.01263 -0.5873702 0.027985274
4-2 -9.8 -18.01263 -1.5873702 0.009769101
7-2 -10.0 -18.21263 -1.7873702 0.007850934
```

There is a significant difference in species diversity between beaches:

- beach 4 and beach 1
- beach 7 and beach 1
- beach 3 and beach 2
- beach 4 and beach 2
- beach 7 and beach 2

Question 5: How does species richness vary with sampling time?
Week $1 \& 2$ versus week $3 \& 4$ t-test (need to create a 2-category variable from 'week").

```
> rikz5 = rikz[rikz$Week < 5,]
> rikz5$fortnight = ifelse(rikz5$Week < 3, 1, 2)
> rikz5$fortnight = as.factor(rikz5$fortnight)
> rikz6 = lm(Richness ~ fortnight, data=rikz5)
> summary(rikz6)
Call:
lm(formula = Richness ~ fortnight, data = rikz5)
Residuals:
    Min 1Q Median 3Q Max
-5.24 -3.24 -1.00 2.00 17.00
Coefficients:
    Estimate Std. Error t value Pr(>|t|)
(Intercept) 6.240 1.004 6.212 1.79e-07 ***
fortnight2 -1.240 1.507 -0.823 0.415
Signif. codes: 0 ،***' 0.001 ،**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
Residual standard error: 5.022 on 43 degrees of freedom
Multiple R-squared: 0.01551, Adjusted R-squared: -0.007387
F-statistic: 0.6774 on 1 and 43 DF, p-value: 0.415
```

The $p$-value of $1.79 e^{-07}$ being the intercept does not mean anything significant. The fact that 0.415 is greater that 0.05 indicates that there is little likelihood that there is any difference between species richness fortnight1 and fortnight2.
Question 6: How does species richness vary with NAP and beach angle?

```
> rikz7 = lm(Richness ~ NAP + angle1, data=rikz)
> summary(rikz7)
Call:
lm(formula = Richness ~ NAP + angle1, data = rikz)
Residuals:
    Min 1Q Median 3Q Max
-4.9890 -3.0331 -0.7893 1.5244 14.0487
Coefficients
    Estimate Std. Error t value Pr(>|t|)
(Intercept) 6.376753 0.903875 7.055 1.21e-08 ***
NAP -2.862905 0.636315 -4.499 5.31e-05 ***
angle1 0.006113 0.012145 0.503 0.617
Signif. codes: 0 '***' 0.001 ،**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
Residual standard error: 4.197 on 42 degrees of freedom
Multiple R-squared: 0.3286, Adjusted R-squared: 0.2966
F-statistic: 10.28 on 2 and 42 DF, p-value: 0.0002327
> rikz8 = lm(Richness ~ NAP + angle2, data=rikz)
> summary(rikz8)
Call:
lm(formula = Richness ~ NAP + angle2, data = rikz)
Residuals:
\begin{tabular}{rrrrr} 
Min & \(1 Q\) & Median & \(3 Q\) & Max \\
-5.4611 & -2.4319 & -0.8159 & 1.4524 & 15.7456
\end{tabular}
Coefficients:
Estimate Std. Error t value Pr(>|t|)
(Intercept) 3.88471 1.34484 2.889 0.00609 **
NAP -2.72332 0.60297 -4.517 5.03e-05 ***
angle2 0.04761 0.02024 2.353 0.02339 *
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
Residual standard error: 3.956 on 42 degrees of freedom
Multiple R-squared: 0.4032, Adjusted R-squared: 0.3748
F-statistic: 14.19 on 2 and 42 DF, p-value: 1.961e-05
```

The $p$-value for angle1 is not significant however the $p$-value of 0.02339 for angle2 demonstrates significance in the difference in species richness.

Question 7: Is there any evidence of an interaction effect between them?

```
> rikz9 = lm(Richness ~ NAP + angle1 + NAP * angle1, data=rikz)
> summary(rikz9)
Call:
lm(formula = Richness ~ NAP + angle1 + NAP * angle1, data = rikz)
Residuals:
Min 1Q Median 3Q Max
-5.3839 -2.7098-0.8666 1.6629 14.6538
Coefficients:
    Estimate Std. Error t value Pr(>|t|)
(Intercept) 5.68941 1.13138 5.029 1.02e-05 ***
NAP -2.13058 0.96476 -2.208 0.0329 *
angle1 0.01947 0.01795 1.084 0.2846
NAP:angle1 -0.01418 0.01404 -1.010 0.3186
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
Residual standard error: 4.196 on 41 degrees of freedom
Multiple R-squared: 0.3449, Adjusted R-squared: 0.2969
F-statistic: 7.195 on 3 and 41 DF, p-value: 0.0005466
> rikz10 = lm(Richness ~ NAP + angle2 + NAP * angle2, data=rikz)
> summary(rikz10)
Call:
lm(formula = Richness ~ NAP + angle2 + NAP * angle2, data = rikz)
Residuals:
        Min 1Q Median 3Q Max
-5.608 -2.265 -1.046 1.453 16.257
Coefficients:
    Estimate Std. Error t value Pr(>|t|)
(Intercept) 3.56005 1.41559 2.515 0.0159 *
NAP -1.81118 1.33015 -1.362 0.1807
angle2 0.05251 0.02131 2.465 0.0180 *
NAP:angle2 -0.01603 0.02081 -0.770 0.4455
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
Residual standard error: 3.976 on 41 degrees of freedom
Multiple R-squared: 0.4117, Adjusted R-squared: 0.3687
F-statistic: 9.565 on 3 and 41 DF, p-value: 6.513e-05
```

There does appear to be a significant impact on species richness when the interaction effect between angle2 and NAP is considered. However there does not appear to be a significant impact when the interaction effect between angle 1 and NAP is considered.

### 11.11 Model Predictions

The predict() function returns predictions from the results of various model fitting functions. Most prediction methods which are similar to those for linear models have an argument newdata specifying the first place to look for explanatory variables to be used for prediction.

Input to the predict() function must be in the format of a dataframe.

```
> owl = read.csv('owl_data.csv')
> owl.lm = lm(begging~sex+food+sex*food, data = owl)
> newdata = data.frame(sex=rep('Male', 20),food=1:20)
> newdata.2 = data.frame(sex=rep('Female', 20),food=1:20)
> owl.lm_male = predict(owl.lm, newdata)
> owl.lm_female = predict(owl.lm, newdata.2)
> owl.lm_male
\rrrrrrrrrrer
9 9
21.56074 20.65260 19.74445 18.83630 17.92816 17.02001 16.11187 15.20372
14.29558 13.38743 12.47929 11.57114
> owl_lm_female
\begin{tabular}{rrrrrrrr}
1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\
22.94928 & 22.23438 & 21.51949 & 20.80459 & 20.08970 & 19.37480 & 18.65991 & 17.94501 \\
9 & 10 & 11 & 12 & 13 & 14 & 15 & 16 \\
17.23012 & 16.51522 & 15.80033 & 15.08543 & 14.37054 & 13.65564 & 12.94075 & 12.22585 \\
17 & 18 & 19 & 20 & & & & \\
11.51096 & 10.79606 & 10.08117 & 9.36627 & & &
\end{tabular}
> colours = c("red","blue", "green",
    "yellow", "purple", "Cyan",
    "pink", "brown"
    )
> title = "Histogram of Owl predictions (Male)"
> title.2 = "Histogram of Owl predictions (Female)"
> hist(owl.lm_male, col = colours, main = title, ylab = "Begging", xlab = "Food")
```

Histogram of Owl predictions (Male)


Illustration 15: owl: Predictions (male)
hist(owl.lm_female, col = colours, main = title.2, ylab = "Begging", xlab = "Food")

Histogram of Owl predictions (Female)


Illustration 16: owl: Predictions (female)

## 12. Distribution models

Thus far the linear modelling assumed that the residuals fell around the line of regression on a Normal or Gaussian distribution. This form of model is a continuous function which approximates the exact binomial distribution of events. The Gaussian distribution shown is normalised so that the sum over all values of $x$ gives a probability of one.

### 12.1 Standard Deviation

A simple example.

```
>a = c(9, 2, 5, 4, 12, 7, 8, 11, 9, 3, 7, 4, 12, 5, 4, 10, 9, 6, 9, 4)
> a
    [1] }90.
>b = mean(a)
> b
[1] 7
>c = a-b
> c
    [1] 2 - 5 - -2 -3 5 5 0 0 1 4 4 2 2 -4 0
>d = c*c
> d
    [1] 4 4 25 4 4 9 25 0
>e=mean(d)
>
[1] 8.9
>f=sqrt(e)
> f
[1] 2.983287
```

So for the set of data $(9,2,5,4,12,7,8,11,9,3,7,4,12,5,4,10,9,6,9,4)$ mean ( $\mu$ ) $=7$ standard deviation $\delta=2.983287$

### 12.1.1 Plotting the Standard Deviation

Note for the plot \# - type: the type of plot to be drawn where "n" means do not plot the points

- xlab: the title of the $x$ axis
- ylab: the title of the $y$ axis
- main: the overall title for the plot
- axes: when FALSE it suppresses the axis automatically generated by the high level plotting function so that we can create custom axis

```
# Set the sample mean to 7 and SD to 2.8
> sample_mean = 7
> sample_sd = 2.8
# Fill one SD
> sd_to_fill = 1
> lower_bound = sample_mean - sample_sd * sd_to_fill
> upper_bound = sample_mean + sample_sd * sd_to_fill
# Generates equally spaced values within 4 SD of mean
> x = seq(-4, 4, length = 1000) * sample_sd + sample_mean
# The height of the probability distribution at each point
> y = dnorm(x, sample_mean, sample_sd)
# Generate the plot
> plot(x, y, type="n", xlab = "Samples", ylab = "", main = "Distribution of
Samples", axes = FALSE)
# Connect the points with each other to form a curve
lines(x, y)
# Returns a vector of boolean values to ensure only x values
# between bounds are allowed
> bounds_filter = x >= lower_bound & x <= upper_bound
> x_within_bounds = x[bounds_filter]
> y_within_bounds = y[bounds_filter]
# Bordering the area to be filled
> x_polygon = c(lower_bound, x_within_bounds, upper_bound)
> y_polygon = c(0, y_within_bounds, 0)
> polygon(x_polygon, y_polygon, col = "green")
# Returns the probability that a normally distributed random number
# will be less than the given number
> probability_within_bounds = pnorm(upper_bound, sample_mean, sample_sd) -
    pnorm(lower_bound, sample_mean, sample_sd)
# Concatenate the various values to display on the curve
    > text = paste("p(", lower_bound, "< height <", upper_bound, ") =",
    signif(probability_within_bounds, digits = 3))
# Display the text on the plot
> mtext(text)>
# Add an axis to the plot
> sd_axis_bounds = 5>
> axis_bounds = seq(-sd_axis_bounds * sample_sd + sample_mean, sd_axis_bounds
    * sample_sd + sample_mean, by = sample_sd)
> axis(side = 1, at = axis_bounds, pos = 0)
```

By changing the value of the sd_fill from one to two and three the various areas under the curve for each standard deviation can be seen. There is a simple rule canned the empirical rule to remember these. $68.27 \%, 95.45 \%$ and $99.73 \%$ of the values lie within one, two and three standard deviations of the mean.


Illustration 17: Standard Deviation

## Another example

Looking at another example in more detail.
To calculate the standard deviation of a set of random numbers:

- Work out the mean ( $\mu$ ).
- For each number, subtract the mean $(x-\mu)$.
- Square the result $(x-\mu)^{2}$.
- Get the mean of the squared differences $1 / \mathrm{N} \Sigma(\mathrm{x}-\mu)^{2}$. This is called the variance (v).
- Get the square root of that $\delta=\sqrt{ } 1 / \mathrm{N} \Sigma(x-\mu)^{2}$.

```
> data.set = round(runif(100,0,1), 2)
> data.set
    [1] 0.56 0.13 0.27 0.91 0.08 0.49 0.99 0.67 0.74 0.59 0.92 0.55 0.18 0.44 0.64
    [16] 0.92 0.15 0.96 0.17 0.67 0.74 0.33 0.52 0.59 0.17 0.59 0.95 0.68 0.71 0.51
    [31] 0.73 0.72 0.71 0.57 0.37 0.73 0.57 0.99 0.84 0.06 0.93 0.16 0.23 0.91 0.75
    [46] 0.89 0.67 0.17 0.29 0.62 0.56 0.35 0.03 0.55 0.62 0.84 0.30 0.69 0.30 0.88
    [61] 0.10 0.03 0.98 0.68 0.38 0.35 0.60 0.82 0.51 0.24 0.58 1.00 0.97 0.65 0.04
    [76] 0.93 0.42 0.97 0.03 0.52 0.38 0.83 0.18 0.37 0.10 0.60 0.96 0.96 0.54 0.65
    [91] 0.75 0.56 0.27 0.94 0.25 0.66 0.41 0.34 0.21 0.99
```

```
> mean_ds = mean(data.set)
> mean_ds
[1] 0.556
```

The mean: $\mu=0.556$
Subtract the Mean from each $x$ to get $(x-\mu)$.

```
> sub ds = data.set - mean ds
```

| > sub_ds |  |  |  |  |  |  |  |  |  |  |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| [1] | 0.004 | -0.426 | -0.286 | 0.354 | -0.476 | -0.066 | 0.434 | 0.114 | 0.184 | 0.034 |
| $[11]$ | 0.364 | -0.006 | -0.376 | -0.116 | 0.084 | 0.364 | -0.406 | 0.404 | -0.386 | 0.114 |
| $[21]$ | 0.184 | -0.226 | -0.036 | 0.034 | -0.386 | 0.034 | 0.394 | 0.124 | 0.154 | -0.046 |
| $[31]$ | 0.174 | 0.164 | 0.154 | 0.014 | -0.186 | 0.174 | 0.014 | 0.434 | 0.284 | -0.496 |
| $[41]$ | 0.374 | -0.396 | -0.326 | 0.354 | 0.194 | 0.334 | 0.114 | -0.386 | -0.266 | 0.064 |
| $[51]$ | 0.004 | -0.206 | -0.526 | -0.006 | 0.064 | 0.284 | -0.256 | 0.134 | -0.256 | 0.324 |
| $[61]$ | -0.456 | -0.526 | 0.424 | 0.124 | -0.176 | -0.206 | 0.044 | 0.264 | -0.046 | -0.316 |
| $[71]$ | 0.024 | 0.444 | 0.414 | 0.094 | -0.516 | 0.374 | -0.136 | 0.414 | -0.526 | -0.036 |
| $[81]$ | -0.176 | 0.274 | -0.376 | -0.186 | -0.456 | 0.044 | 0.404 | 0.404 | -0.016 | 0.094 |
| $[91]$ | 0.194 | 0.004 | -0.286 | 0.384 | -0.306 | 0.104 | -0.146 | -0.216 | -0.346 | 0.434 |

Square each result to get $(x-\mu)^{2}$.

```
> sq_ds = sub_ds * sub_ds
> sq_ds
    [1] 0.000016 0.181476 0.081796 0.125316 0.226576 0.004356 0.188356 0.012996
    [9] 0.033856 0.001156 0.132496 0.000036 0.141376 0.013456 0.007056 0.132496
[17] 0.164836 0.163216 0.148996 0.012996 0.033856 0.051076 0.001296 0.001156
[25] 0.148996 0.001156 0.155236 0.015376 0.023716 0.002116 0.030276 0.026896
[33] 0.023716 0.000196 0.034596 0.030276 0.000196 0.188356 0.080656 0.246016
[41] 0.139876 0.156816 0.106276 0.125316 0.037636 0.111556 0.012996 0.148996
[49] 0.070756 0.004096 0.000016 0.042436 0.276676 0.000036 0.004096 0.080656
[57] 0.065536 0.017956 0.065536 0.104976 0.207936 0.276676 0.179776 0.015376
[65] 0.030976 0.042436 0.001936 0.069696 0.002116 0.099856 0.000576 0.197136
[73] 0.171396 0.008836 0.266256 0.139876 0.018496 0.171396 0.276676 0.001296
[81] 0.030976 0.075076 0.141376 0.034596 0.207936 0.001936 0.163216 0.163216
[89] 0.000256 0.008836 0.037636 0.000016 0.081796 0.147456 0.093636 0.010816
[97] 0.021316 0.046656 0.119716 0.188356
```

Get the mean of these values $1 / N \Sigma(x-\mu)^{2}$.

```
> mean_ds_sd = mean(sq_ds)
> mean_ds_sd
[1] 0.081938
```

Therefore $\delta=0.081938$.

### 12.2 Expand Grid - expand.grid()

Create a data-frame from all combinations of the supplied vectors or factors.

| > grid.data |  |  |  |
| :---: | :---: | :---: | :---: |
|  |  | weight | sex |
| 1 | 60 | 100 | Male |
| 2 | 65 | 100 | Male |
| 3 | 70 | 100 | Male |
| 4 | 75 | 100 | Male |
| 5 | 80 | 100 | Male |
| 6 | 60 | 150 | Male |
| 7 | 65 | 150 | Male |
| 8 | 70 | 150 | Male |
| 9 | 75 | 150 | Male |
| 10 | 80 | 150 | Male |
| 11 | 60 | 200 | Male |
| 12 | 65 | 200 | Male |
| 13 | 70 | 200 | Male |
| 14 | 75 | 200 | Male |


| 15 | 80 | 200 | Male |
| :---: | :---: | :---: | :---: |
| 16 | 60 | 250 | Male |
| 17 | 65 | 250 | Male |
| 18 | 70 | 250 | Male |
| 19 | 75 | 250 | Male |
| 20 | 80 | 250 | Male |
| 21 | 60 | 300 | Male |
| 22 | 65 | 300 | Male |
| 23 | 70 | 300 | Male |
| 24 | 75 | 300 | Male |
| 25 | 80 | 300 | Male |
| 26 | 60 | 100 | Female |
| 27 | 65 | 100 | Female |
| 28 | 70 | 100 | Female |
| 29 | 75 | 100 | Female |
| 30 | 80 | 100 | Female |
| 31 | 60 | 150 | Female |
| 32 | 65 | 150 | Female |
| 33 | 70 | 150 | Female |
| 34 | 75 | 150 | Female |
| 35 | 80 | 150 | Female |
| 36 | 60 | 200 | Female |
| 37 | 65 | 200 | Female |
| 38 | 70 | 200 | Female |
| 39 | 75 | 200 | Female |
| 40 | 80 | 200 | Female |
| 41 | 60 | 250 | Female |
| 42 | 65 | 250 | Female |
| 43 | 70 | 250 | Female |
| 44 | 75 | 250 | Female |
| 45 | 80 | 250 | Female |
| 46 | 60 | 300 | Female |
| 47 | 65 | 300 | Female |
| 48 | 70 | 300 | Female |
| 49 | 75 | 300 | Female |
| 50 | 80 | 300 | Female |

### 12.3 Normal or Gaussian distribution

What has been shown is actually a Normal or Gaussian distribution. This is a very common continuous probability distribution that are often used to represent real-valued random variables whose distributions are not known.

### 12.4 Poisson distribution

The Poisson distribution is the probability distribution of independent event occurrences in an interval. If lambda $(\lambda)$ is the mean occurrence per interval, then the probability of having $x$ occurrences within a given interval is:
$f(x)=\lambda^{x} e^{-\lambda} / x$ ! where $x=0,1,2,3, n$


Note that the Poisson distribution is for zero and positive values only. This can be useful if the dataset being operated on cannot have negative values.
For example.
Question: If there are five birds spotted landing on a pond per minute on average, what is the probability of eight birds landing on the pond in any minute?
The probability of having eight or less birds landing on the pond in any minute is given by the ppois() function. This function is the density, distribution function, quantile function and random generation for the Poisson distribution with parameter lambda ( $\lambda$ ). This is called the lower tail of the function.

```
> ppois(8, lambda=5)
[1] 0.9319064
```

Secondly it is necessary to calculate the probability of having eight or more birds landing on the pond in any minute is called the upper tail. Hence the probability of having seventeen or more cars crossing the bridge in a minute is in the upper tail of the probability density function.

```
> ppois(8, lambda=5, lower=FALSE)
[1] 0.06809363
```

So there is a $93 \%$ chance of having eight or less birds landing on the pond while there is a $6.8 \%$ chance of having eight or more birds landing on the pond in and minute.

### 12.4.1 Linear Models based on Poisson Distribution

Returning to an earlier example.

```
> setwd('~/datasets/RIKZ_2')
> rikz = read.csv('RIKZ.csv')
> rikz10 = lm(Richness ~ NAP + angle2 + NAP * angle2, data=rikz)
```

```
> summary(rikz10)
Call:
lm(formula = Richness ~ NAP + angle2 + NAP * angle2, data = rikz)
Residuals:
    Min 1Q Median 3Q Max
-5.608 -2.265 -1.046 1.453 16.257
Coefficients:
    Estimate Std. Error t value Pr(>|t|)
(Intercept) 3.56005 1.41559 2.515 0.0159 *
NAP -1.81118 1.33015 -1.362 0.1807
angle2 0.05251 0.02131 2.465 0.0180 *
NAP:angle2 -0.01603 0.02081 -0.770 0.4455
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
Residual standard error: 3.976 on 41 degrees of freedom
Multiple R-squared: 0.4117, Adjusted R-squared: 0.3687
F-statistic: 9.565 on 3 and 41 DF, p-value: 6.513e-05
```

Using the Normal Distribution did not show a p-value of much significance.
Try running a linear model using the Poisson Distribution. In $R$ to do this it is necessary to run the Fitting Generalized Linear Models which offers the family option. to run models based on other distributions like poisson.

```
> rikz11 = glm(Richness ~ NAP + angle2 + NAP * angle2, data=rikz, family=poisson)
> summary(rikz11)
Call:
glm(formula = Richness ~ NAP + angle2 + NAP * angle2, family = poisson,
    data = rikz)
Deviance Residuals:
\begin{tabular}{rrrrr} 
Min & \(1 Q\) & Median & \(3 Q\) & Max \\
-2.4543 & -1.1122 & -0.7233 & 0.7948 & 5.0552
\end{tabular}
Coefficients:
    Estimate Std. Error z value Pr(>|z|)
(Intercept) 1.259472 0.157057 8.019 1.06e-15 ***
NAP -0.743224 0.188056 -3.952 7.75e-05 ***
angle2 0.008586 0.002176 3.946 7.95e-05 ***
NAP:angle2 0.003209 0.002523 1.272 0.203
Signif. codes: 0 '***' 0.001 ‘**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
(Dispersion parameter for poisson family taken to be 1)
    Null deviance: 179.753 on 44 degrees of freedom
Residual deviance: 97.026 on 41 degrees of freedom
AIC: 247.02
Number of Fisher Scoring iterations: 5
```

As can be seen the p -values show much more significance for this distribution model.

### 12.4.2 Linear Models based on other Distributions

There are many other distribution models. Here is a summary and $R$ offers the capability to run tests with each by adjusting the family value.

| Continuous Data | Data range | Link Function |
| :---: | :--- | :---: |
| Normal | Any value, positive or negative | 1 |
| Gamma | Positive values only | log |
| Beta | Values between zero and one | logit |
| Discrete Data | Data range | Link Function |
| Poisson | Zero and positive values | log |
| Binomial | Zero and positive values | logit |

```
> rikz12 = glm(Richness ~ NAP, data=rikz, family=binomial)
Error in eval(family$initialize) : y values must be 0 <= y <= 1
```

In this case //Richness/ would have to be yes/no values.

```
> rikz13 = glm(Richness ~ NAP, data=rikz, family=gamma)
Error in family() : 0 arguments passed to 'gamma' which requires 1
```

In this case Richness would have to consist of count values.

### 12.4.3 Link Functions

The link function provides the relationship between the linear predictor and the mean of the distribution function. There are many commonly used link functions, and their choice is informed by several considerations. The table above gives the link functions associated with the various distributions.

### 12.4.4 Type Response

The type of prediction is required for predict.glm. The default is on the scale of the linear predictors; the alternative response is on the scale of the response variable. Thus for a default binomial model the default predictions are of log-odds (probabilities on logit scale) and type $=$ "response" gives the predicted probabilities. The terms option returns a matrix giving the fitted values of each term in the model formula on the linear predictor scale.
The alternative type to response is the type = "terms" option which returns a matrix giving the fitted values of each term in the model formula on the linear predictor scale.

```
> owl = read.csv('owl_data.csv')
> owl.glm = glm(begging~sex+food+sex*food, family=poisson, data = owl)
> newdata = data.frame(sex=rep('Male',20),food=1:20)
> owl.glm_male = predict.glm(owl.glm, newdata, family=poisson)
```

```
\begin{tabular}{rrrrrrrr} 
> owl.glm_male & & & & \\
1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\
5.155740 & 5.020808 & 4.885877 & 4.750945 & 4.616013 & 4.481082 & 4.346150 & 4.211218 \\
9 & 10 & 11 & 12 & 13 & 14 & 15 & 16 \\
4.076286 & 3.941355 & 3.806423 & 3.671491 & 3.536559 & 3.401628 & 3.266696 & 3.131764 \\
17 & 18 & 19 & 20 & & & & \\
2.996832 & 2.861901 & 2.726969 & 2.592037 & & & &
\end{tabular}
> owl.glm_male = predict.glm(owl.glm, newdata, family=poisson, type='response')
> owl.glm_male
\begin{tabular}{rrrrrrr}
1 & 2 & 3 & 4 & 5 & 6 & 7 \\
173.42413 & 151.53377 & 132.40650 & 115.69357 & 101.09021 & 88.33015 & 77.18073 \\
9 & 10 & 11 & 12 & 13 & 14 & 15
\end{tabular}
> owl.glm_male = predict.glm(owl.glm, newdata, family=poisson, type='terms')
> owl.glm_male
    sex food sex:food
    0.1482603 3.0438445 0.093027486
    0.1482603 2.9157238 0.086216467
    0.1482603 2.7876031 0.079405448
    0.1482603 2.6594824 0.072594430
    0.1482603 2.5313617 0.065783411
    0.1482603 2.4032409 0.058972392
    0.1482603 2.2751202 0.052161374
    0.1482603 2.1469995 0.045350355
    0.1482603 2.0188788 0.038539336
    0.1482603 1.8907581 0.031728318
    11 0.1482603 1.7626373 0.024917299
    12 0.1482603 1.6345166 0.018106280
    13 0.1482603 1.5063959 0.011295261
    14 0.1482603 1.3782752 0.004484243
    15 0.1482603 1.2501545 -0.002326776
    16 0.1482603 1.1220338-0.009137795
    17 0.1482603 0.9939130 -0.015948813
    18 0.1482603 0.8657923-0.022759832
19 0.1482603 0.7376716-0.029570851
20 0.1482603 0.6095509 -0.036381869
attr(,"constant")
[1] 1.870608
```


### 12.5 Exercise: Linear Modelling 1

Using the insect_spray.csv data which shows counts of insects after a particular type of insect spray has been used.

1. Run a simple linear model comparing the effectiveness of the different sprays (this will be a traditional ANOVA).
2. Get predictions and their standard errors from this model for each spray type (using the predict.Im function).
3. Run the model again as a GLM using the poisson distribution (are the estimates different from model 1 ? why are they so different?).
4. Get predictions and their standard errors (using the predict.glm function). How do the SEs compare to model 1. Why are they so different?
5. Create a matrix to store the predictions and SEs from both models (i.e. four column matrix).
6. Save this matrix table of results as a .csv file in your working directory.

## Answer:

Import the data.

```
> insect_spray = read.csv('insect_spray.csv')
> names(insect_spray)
[1] "X" "count" "spray"
> head(insect_spray)
    X count spray
11010 A
\begin{tabular}{lll}
2 & 2 & 7
\end{tabular}
3320 A
4414 A
5514 A
6612 A
> str(insect_spray)
'data.frame': 72 obs. of 3 variables:
    $ X : int 1 2 3 4 5 6 7 8 9 10 ...
    $ count: int 10 7 20 14 14 12 10 23 17 20 \ldots..
    $ spray: Factor w/ 6 levels "A","B","C","D",..: 1 1 1 1 1 1 1 1 1 1 ..
> summary(insect_spray)
\begin{tabular}{|c|c|c|}
\hline X & count & spray \\
\hline Min. : 1.00 & Min. : 0.00 & A:12 \\
\hline 1st Qu.:18.75 & 1st Qu.: 3.00 & B:12 \\
\hline Median :36.50 & Median : 7.00 & C:12 \\
\hline Mean :36.50 & Mean : 9.50 & D:12 \\
\hline 3rd Qu.:54.25 & 3rd Qu.:14.25 & E:12 \\
\hline Max. :72.00 & Max. :26.00 & F:12 \\
\hline
\end{tabular}
```

Run a simple linear model comparing the effectiveness of the different sprays.

```
> mod1 = lm(count~spray, data=insect_spray)
> summary(mod1)
Call:
lm(formula = count ~ spray, data = insect_spray)
Residuals:
    Min 1Q Median 3Q Max
-8.333 -1.958 -0.500 1.667 9.333
Coefficients:
    Estimate Std. Error t value Pr(>|t|)
(Intercept) 14.5000 1.1322 12.807 < 2e-16 ***
\begin{tabular}{lllll} 
sprayB & 0.8333 & 1.6011 & 0.520 & 0.604
\end{tabular}
sprayC -12.4167 1.6011 -7.755 7.27e-11 ***
sprayD -9.5833 1.6011 -5.985 9.82e-08 ***
sprayE -11.0000 1.6011 -6.870 2.75e-09 ***
sprayF 2.1667 1.6011 \(1.353 \quad 0.181\)
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Residual standard error: 3.922 on 66 degrees of freedom
Multiple R-squared: 0.7244, Adjusted R-squared: 0.7036
F-statistic: 34.7 on 5 and 66 DF, $p$-value: < 2.2e-16

```
> anova(mod1)
Analysis of Variance Table
Response: count
    Df Sum Sq Mean Sq F value Pr(>F)
spray 5 2668.8 533.77 34.702 < 2.2e-16 ***
Residuals 66 1015.2 15.38
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Get predictions and their standard errors from this model for each spray type.

```
> new.data = data.frame(spray=c("A", "B", "C", "D", "E", "F"))
> pred1 = predict.lm(mod1, new.data, se.fit=T)
> summary(pred1)
    Length Class Mode
fit 6 -none- numeric
se.fit 6 -none- numeric
df 1 -none- numeric
residual.scale 1 -none- numeric
> pred1
$fit
\begin{tabular}{rrrrrr}
1 & 2 & 4 & 5 & 6 \\
14.500000 & 15.333333 & 2.083333 & 4.916667 & 3.500000 & 16.666667
\end{tabular}
$se.fit
\begin{tabular}{rrrrrr}
1 & 2 & 3 & 4 & 5 & 6
\end{tabular}
$df
[1] 66
```

\$residual.scale
[1] 3.921902

Run the model again as a GLM using the poisson distribution. Are the estimates different from model one?

```
> mod2 = glm(count~spray, data=insect_spray, family=poisson)
> summary(mod2)
Call:
glm(formula = count ~ spray, family = poisson, data = insect_spray)
Deviance Residuals:
\begin{tabular}{rrrrr} 
Min & \(1 Q\) & Median & 3Q & Max \\
-2.3852 & -0.8876 & -0.1482 & 0.6063 & 2.6922
\end{tabular}
```

Coefficients:
Estimate Std. Error z value $\operatorname{Pr}(>|z|)$
(Intercept) $2.67415 \quad 0.07581 \quad 35.274<2 \mathrm{e}-16$ ***
sprayB $0.05588 \quad 0.10574 \quad 0.528 \quad 0.597$
sprayC -1.94018 0.21389-9.071 < 2e-16 ***
sprayD -1.08152 0.15065 -7.179 7.03e-13 ***
sprayE -1.42139 $0.17192-8.268<2 e-16$ ***

| sprayF | 0.13926 | 0.10367 | 1.343 | 0.179 |
| :--- | :--- | :--- | :--- | :--- |

Signif. codes: $0{ }^{\prime * * * ' ~} 0.001$ ‘**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
(Dispersion parameter for poisson family taken to be 1)
Null deviance: 409.041 on 71 degrees of freedom
Residual deviance: 98.329 on 66 degrees of freedom

```
AIC: 376.59
Number of Fisher Scoring iterations: 5
> anova(mod2)
Analysis of Deviance Table
Model: poisson, link: log
Response: count
Terms added sequentially (first to last)
    Df Deviance Resid. Df Resid. Dev
NULL \(\quad 71\)\begin{tabular}{rrr} 
& 409.04
\end{tabular}
\begin{tabular}{llll} 
spray 5 & 310.71 & 66 & 98.33
\end{tabular}
```

Get predictions and their standard errors (using the predict.glm function). How do the SEs compare to model one. Why are they so different?

```
> new.data = data.frame(spray=c("A", "B", "C", "D", "E", "F"))
> pred2 = predict.glm(mod2, new.data, se.fit=T, type="response")
> summary(pred2)
    Length Class Mode
fit 6 -none- numeric
se.fit 6 -none- numeric
residual.scale 1 -none- numeric
> pred2
$fit
14.500000 15.333333 2.083333 4.916667 3.500000 16.666667
$se.fit
    1 2 < 3 4 4 % 5
1.0992422 1.1303883 0.4166664 0.6400955 0.5400617 1.1785113
```

\$residual.scale
[1] 1

Save this matrix table of results as a .csv file in your working directory.

```
> store = matrix(0, nrow=6, ncol=4)
> store[,1] = pred1$fit
> store[,2] = pred1$se.fit
> store[,3] = pred2$fit
> store[,4] = pred2$se.fit
> write.csv(store, "insect_pred_table.csv")
> quit()
Save workspace image? [y/n/c]: n
$ cat insect_pred_table.csv
"","V1","V2","V3","V4"
"1",14.5,1.13215550799177,14.5,1.0992421631893
"2",15.3333333333333,1.13215550799177,15.3333333333333,1.13038833052086
"3", 2.08333333333331,1.13215550799177,2.08333333333498,0.416666404601133
"4",4.91666666666668,1.13215550799177,4.91666666666666,0.640095478972524
"5",3.50000000000001,1.13215550799177,3.49999999999999,0.540061724814324
"6",16.6666666666667,1.13215550799177,16.6666666666667,1.17851130197669
```


### 12.6 Exercise: Linear Modelling 2

Using the CO2.csv data which shows the uptake of carbon dioxide of two plant types when kept at different temperatures and at different concentrations of $\mathrm{CO}_{2}$.

1. Run a simple linear model (regression) comparing the uptake of $\mathrm{CO}_{2}$ to the concentration of $\mathrm{CO}_{2}$.
2. Run a simple linear model (t-test) comparing the effect of temperature on $\mathrm{CO}_{2}$ uptake.
3. Combine these two explanatory variables into a model that includes them both, and also look at one that includes an interaction between them.
4. Get predictions for the two types of plants, for a range of $\mathrm{CO}_{2}$ concentrations.
5. Rerun the models and get predictions for a Gamma distribution.
6. Compare the different models and find the best model by looking at their Akaike Information Criterion (AIC).

Note: The AIC is an estimator of the relative quality of statistical models for a given set of data. Given a collection of models for the data, AIC estimates the quality of each model, relative to each of the other models. Thus, AIC provides a means for model selection. The AIC can be used to help decide which model is better at describing your data and making predictions from it. The lower the AIC, the better the model.
HINT: an example of using the function is $\operatorname{AIC}(\bmod 1, \bmod 2, \bmod 3)$.

## Answer:

Import the data.

```
> co2 = read.csv('CO2.csv')
> names(co2)
[1] "X" "Plant" "Type" "Treatment" "conc" "uptake"
> head(co2)
    X Plant Type Treatment conc uptake
    11 Qn1 Quebec nonchilled 95 16.0
2 2 Qn1 Quebec nonchilled 175 30.4
3 3 Qn1 Quebec nonchilled 250 34.8
4 4 Qn1 Quebec nonchilled 350 37.2
5 Qn1 Quebec nonchilled 500 35.3
6 6 Qn1 Quebec nonchilled 675 39.2
> str(co2)
'data.frame': 84 obs. of 6 variables:
    $ X : int 1 2 3 4 5 6 7 8 9 10 ...
    $ Plant : Factor w/ 12 levels "Mc1","Mc2","Mc3",..: 10 10 10 10 10 10 10 11 11
11 ...
    $ Type : Factor w/ 2 levels "Mississippi",..: 2 2 2 2 2 2 2 2 2 2 ...
    $ Treatment: Factor w/ 2 levels "chilled","nonchilled": 2 2 2 2 2 2 2 2 2 2 ...
    $ conc : int 95 175 250 350 500 675 1000 95 175 250 ...
    $ uptake: num 16 30.4 34.8 37.2 35.3 39.2 39.7 13.6 27.3 37.1 ...
```

```
> summary(co2)
    X
Min. : 1.00
1st Qu.:21.75
Median :42.50
Mean :42.50
3rd Qu.:63.25
Max. :84.00 Mn3 : 7
(Other):42
        uptake
    Min. : 7.70
1st Qu.:17.90
Median :28.30
Mean :27.21
3rd Qu.:37.12
Max. :45.50
\begin{tabular}{|c|}
\hline > summary(co2) \\
\hline X \\
\hline Min. : 1.00 \\
\hline 1st Qu.:21.75 \\
\hline Median :42.50 \\
\hline Mean :42.50 \\
\hline 3rd Qu.:63.25 \\
\hline Max. \(: 84.00\) \\
\hline uptake \\
\hline Min. : 7.70 \\
\hline 1st Qu.:17.90 \\
\hline Median :28.30 \\
\hline Mean :27.21 \\
\hline 3rd Qu.:37.12 \\
\hline Max. 445.50 \\
\hline
\end{tabular}
```

Run a simple linear model (regression) comparing the uptake of $\mathrm{CO}_{2}$ to the concentration of $\mathrm{CO}_{2}$.

```
> mod1 = lm(uptake~conc, data=co2)
> summary(mod1)
Call:
lm(formula = uptake ~ conc, data = co2)
Residuals:
    Min 1Q Median 3Q Max
-22.831 -7.729 1.483 7.748 16.394
Coefficients
    Estimate Std. Error t value Pr(>|t|)
(Intercept) 19.500290 1.853080 10.523 < 2e-16 ***
conc 0.017731 0.003529 5.024 2.91e-06 ***
Signif. codes: 0 '***' 0.001 ،**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
Residual standard error: 9.514 on 82 degrees of freedom
Multiple R-squared: 0.2354, Adjusted R-squared: 0.2261
F-statistic: 25.25 on 1 and 82 DF, p-value: 2.906e-06
```

Run a simple linear model ( $t$-test) comparing the effect of temperature on $\mathrm{CO}_{2}$ uptake.

```
> mod2 = lm(uptake~Treatment, data=co2)
> summary(mod2)
Call:
lm(formula = uptake ~ Treatment, data = co2)
Residuals:
\begin{tabular}{rrrrr} 
Min & \(1 Q\) & Median & \(3 Q\) & Max \\
-20.0429 & -8.6530 & -0.4429 & 9.7321 & 18.6167
\end{tabular}
```

Coefficients:


Residual standard error: 10.31 on 82 degrees of freedom Multiple R-squared: 0.1018, Adjusted R-squared: 0.09084 F-statistic: 9.293 on 1 and 82 DF, p-value: 0.003096

Combine these two explanatory variables into a model that includes them both, and also look at one that includes an interaction between them.

```
> mod3a = lm(uptake~conc + Treatment, data=co2)
>mod3b = lm(uptake~conc * Treatment, data=co2)
> summary(mod3a)
Call:
lm(formula = uptake ~ conc + Treatment, data = co2)
Residuals:
    Min 1Q Median 3Q Max
-19.401 -7.066 -1.168 7.573 17.597
Coefficients:
```



```
Residual standard error: 8.912 on 81 degrees of freedom
Multiple R-squared: 0.3372, Adjusted R-squared: 0.3208
F-statistic: 20.6 on 2 and 81 DF, p-value: 5.837e-08
> summary(mod3b)
Call:
lm(formula = uptake ~ conc * Treatment, data = co2)
Residuals:
\begin{tabular}{rrrrr} 
Min & \(1 Q\) & Median & \(3 Q\) & Max \\
-18.218 & -7.401 & -1.117 & 7.835 & 17.209
\end{tabular}
Coefficients:
\begin{tabular}{lrrrrr} 
& Estimate Std. Error t value \(\operatorname{Pr}(>|\mathrm{t}|)\) \\
(Intercept) & 16.981416 & 2.464160 & 6.891 & \(1.15 \mathrm{e}-09\)
\end{tabular} ***
Signif. codes: 0 '***'' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
Residual standard error: 8.946 on 80 degrees of freedom
Multiple R-squared: 0.3405, Adjusted R-squared: 0.3157
F-statistic: 13.77 on 3 and 80 DF, p-value: 2.528e-07
```

Get predictions for the two types of plants, for a range of $\mathrm{CO}_{2}$ concentrations.

```
> mod4 = lm(uptake~conc * Type, data=co2)
> new.data = expand.grid(Type=c("Quebec", "Mississippi"),
    conc=seq(100,1000,100)
    )
```

```
> predict.lm(mod4, new.data, se.fit=T)
$fit
\begin{tabular}{rrrrrrrr}
1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\
25.81104 & 16.73566 & 28.11905 & 17.97377 & 30.42705 & 19.21188 & 32.73506 & 20.44999 \\
9 & 10 & 11 & 12 & 13 & 14 & 15 & 16 \\
35.04306 & 21.68811 & 37.35106 & 22.92622 & 39.65907 & 24.16433 & 41.96707 & 25.40245 \\
17 & 18 & 19 & 20 & & & & \\
44.27508 & 26.64056 & 46.58308 & 27.87867 & & &
\end{tabular}
$se.fit
\begin{tabular}{rrrrrrr}
1 & 2 & 3 & 4 & 5 & 6 & 7 \\
1.622000 & 1.622000 & 1.369811 & 1.369811 & 1.177546 & 1.177546 & 1.077770 \\
9 & 10 & 11 & 12 & 13 & 14 & 15 \\
1.096037 & 1.096037 & 1.227089 & 1.227089 & 1.440463 & 1.440463 & 1.705538 \\
17 & 18 & 19 & 20 & 1.705538 \\
\hline 2.001880 & 2.001880 & 2.317526 & 2.317526 & & & \\
\hline
\end{tabular}
$df
[1] }8
$residual.scale
[1] 6.935822
```

Rerun the models and get predictions for a Gamma distribution.

```
> mod5 = glm(uptake~conc * Type, data=co2, family=Gamma)
> summary(mod5)
Call:
glm(formula = uptake ~ conc * Type, family = Gamma, data = co2)
Deviance Residuals:
\begin{tabular}{rrrrr} 
Min & \(1 Q\) & Median & \(3 Q\) & Max \\
-0.90639 & -0.22457 & -0.01455 & 0.19478 & 0.54514
\end{tabular}
Coefficients:
                Estimate Std. Error t value Pr(>|t|)
(Intercept) 6.020e-02 4.361e-03 13.803 < 2e-16 ***
conc -2.532e-05 6.898e-06 -3.671 0.000434 ***
TypeQuebec -2.148e-02 5.156e-03 -4.166 7.78e-05 ***
conc:TypeQuebec 7.311e-06 8.094e-06 0.903 0.369081
Signif. codes: 0 ،***' 0.001 ‘**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
(Dispersion parameter for Gamma family taken to be 0.0939836)
    Null deviance: 15.8858 on 83 degrees of freedom
Residual deviance: 8.4588 on 80 degrees of freedom
AIC: 596.1
Number of Fisher Scoring iterations: 5
> new.data = expand.grid(Type=c("Quebec", "Mississippi"),
conc=seq(100,1000,100)
)
```

```
> predict.glm(mod5, new.data, se.fit=T, type="response")
$fit
\begin{tabular}{rrrrrrr}
1 & 2 & 3 & 4 & 5 & 6 & 7 \\
27.08678 & 17.34099 & 28.47595 & 18.13741 & 30.01533 & 19.01050 & 31.73064 \\
9 & 10 & 11 & 12 & 13 & 14 & 15 \\
33.65390 & 21.03572 & 35.82534 & 22.21925 & 38.29633 & 23.54390 & 41.13343 \\
17 & 18 & 19 & 20 & & & 16 \\
44.42452 & 26.73116 & 48.28806 & 28.67189 & & &
\end{tabular}
\begin{tabular}{rrrrrrrr} 
\$se.fit & 2 & 3 & 4 & 5 & 6 & 7 & 8
\end{tabular}
1.7561008 1.1382195 1.6721912 1.0702451 1.5951945 1.0088967 1.5520705 0.9732649
```



```
17 18 19 20
3.9288835 2.3729936 5.3967264 3.1730722
$residual.scale
[1] 0.3065674
```

Compare the different models and find the best model by looking at their AIC.

```
> AIC(mod1, mod2, mod3a, mod3b, mod4, mod5)
    df AIC
mod1 3 620.8180
mod2 3 634.3456
mod3a 4 610.8169
mod3b 5 612.3997
mod4 5 569.6488
mod5 5 596.0955
```


## 13. Plots

Simple plot. plot() is a generic base graphic function for the plotting of $R$ objects. $x$ and $y$ objects must be given to the function.
$>x=1: 5$
$>y=1: 5$
$>\operatorname{plot}(x, y)$


Illustration 19: Simple plot
$\operatorname{plot}(x=c(1,2,3,4), y=c(1,2,3,4))$
$>\operatorname{args}(p l o t)$
function (x, y, ...)
NULL

### 13.1 Beautify the plot

Add type to adjust the view of the plot. What type of plot should be drawn. Possible types are:

| $\mathbf{p}$ | points |
| :--- | :--- |
| $\mathbf{I}$ | lines |
| $\mathbf{b}$ | both |

c the lines part alone of $\mathbf{b}$
o both over|plotted
h histogram like (or high|density) vertical lines
s stair steps
n no plotting
> $x=1: 5$
$>y=1: 5$
> plot( $x, y$, type $=" b ")$


Illustration 20: Plot with lines

Add some colour.
> plot(x,y, type = "b", col = "green")


Illustration 21: Plot with colour

It also works as a general plot function for many object types. See Illustration 22 for a simple linear regression.

```
> x = c(1,2,2,3,2,3,4,3,4,5,3)
>y = c(4,8,6,3,5,7,9,2,1,7,4)
> model.1 = lm(y~x)
> class(model.1)
[1] "lm"
> plot(model.1)
```



Illustration 22: Plot - model1

### 13.1.1 Plot layers

Add some other values.

```
> plot(x,y, type = "b", col = "green",xlab = "bottom", ylab = "left")
> title(main = "My plot", xlab = "length", ylab = "height")
```



Illustration 23: Plot layers

Notice how each or these lines layered on to of each-other. It is necessary to turn off some things in the first line.

```
> plot(x,y, type = "b", col = "green",xlab = "", ylab = "", xaxt="n", xaxt="n")
> title(main = "My plot", xlab = "length", ylab = "height")
```



Illustration 24: Plot layers 2

### 13.1.2 Box type

Use the argument box type [bty] to adjust the box.

- $\mathbf{o}$ : The default value draws a complete rectangle around the plot.
- n: Draws nothing around the plot.
- I, 7, c, u, or ]: Draws a shape around the plot area that resembles the uppercase letter of the option. So, the option bty="I" draws a line to the left and bottom of the plot.

```
> plot(rnorm(100))
> plot(rnorm(100),bty="o")
> plot(rnorm(100),bty="l")
> plot(rnorm(100),bty="7")
> plot(rnorm(100),bty="c")
> plot(rnorm(100),bty="]")
plot(rnorm(100),bty="n")
```



Illustration 25: Box type

### 13.1.3 Splitting arguments

```
> plot(x,y, xlab="height", ylab="length", cex.axis= 1.2,
    cex.lab=1.5, typ="p", col="red", bty="l", pch=16,
        tck=0.03
        )
> plot(x,y, # call the plot
    xlab="height", ylab="length", # label the axes
    xlim=c(0,10), # set specific limits to x-axis
    cex.axis=1.2, cex.lab=1.5, # set character size for axis & labels
    typ="p", col="red", # set plot type & colour
    bty="l", # set box type around plot
    pch=16, # set point character type
    tck=0.03 # set axis tick marks (+ve is inside plot)
    )
```



Illustration 26: Plot - splitting arguments

### 13.1.4 Graphical parameters: par

par(): can be used to set or query graphical parameters. Parameters can be set by specifying them as arguments to par in tag = value form, or by passing them as a list of tagged values. par() on its own returns the current settings for default graphical parameters. These defaults can be modified in par(). Some graphical parameters must be set in par() like background colour.

```
> par(lty=2, # Set the line type
    pch=17, # Define the plotting symbol
    cex.axis=3 # Specify the size of the tick labels
    )
```


## Icon type: PCH

Sets the icon type.

| pch $=0$ | square | pch $=13$ | circle cross |
| :--- | :--- | :--- | :--- |
| pch $=1$ | circle | pch $=14$ | square and triangle down |
| pch $=2$ | triangle point up | pch $=15$ | filled square |
| pch $=3$ | plus | pch $=16$ | filled circle |
| pch $=4$ | cross | pch $=17$ | filled triangle point up |
| pch $=5$ | diamond | pch $=18$ | filled diamond |
| pch $=6$ | triangle point down | pch $=19$ | solid circle up |
| pch $=7$ | square cross | pch $=20$ | bullet (smaller circle) |
| pch $=8$ | star | pch $=21$ | filled circle blue |
| pch $=9$ | diamond plus | pch $=22$ | filled square blue |
| pch $=10$ | circle plus | pch $=23$ | filled diamond blue |
| pch $=11$ | triangles up and down | pch $=24$ | filled triangle point up blue |
| pch $=12$ | square plus | pch $=25$ | filled triangle point down blue |

### 13.1.5 Colours

$R$ has 657 colours to choose from. The colours() function gives a list of the available colour names.
colours(): Creates a list of all available colours.

```
colours = colours()
    > head(colours)
    [1] "white" "aliceblue" "antiquewhite" "antiquewhite1"
    [5] "antiquewhite2" "antiquewhite3"
```

rainbow(n): Creates a vector of $n$ contiguous colors.

```
> rainbow = rainbow(5)
> rainbow
[1] "#FF0000FF" "#CCFF00FF" "#00FF66FF" "#0066FFFF" "#CC00FFFF"
```

grey(level): Creates a vector of colours from a vector of gray levels. These are given as a vector of desired gray levels between zero and one; zero indicates black and one indicates white.

```
> grey = grey(c(0.1,0.3,0.5,0.7,0.9))
> grey
[1] "#1A1A1A" "#4D4D4D" "#808080" "#B3B3B3" "#E6E6E6"
```


### 13.1.6 text

text(): Insert text at any position of a current open plot.

```
> text(x, y, # x,y plot co-ordinates for the text
    labels, # Text to be inserted
    pos, # Position (1,2,3,4=below,left,above,right)
    offset, # Distance of pos offset from x,y
    col, cex, font ... # Other options
    )
```

Example:

```
> plot(1,1)
> text(1,1.05,
    "This is a dot in the middle",
    col="red",cex=0.8
        )
> text(1,1,
    "This is the test to be printed",
        pos=1,offset=1,col="red", cex=0.8
        )
```



Illustration 27: Plot text

Another example.

```
> plot(0,1,xaxt='n',yaxt='n')
> my_text = list(bquote( paste( "Average Area=5.78" , m^2/h) ),
    bquote( paste( "Average Distance=12.2", km/h ) )
    )
>
> mtext(side=1,do.call(expression, my_text), line=-1:-2, adj=0)
```

$\square$

0

Illustration 28: Plot text 2

### 13.1.7 Points

points(): Insert points at any position of a current open plot.

```
points(x, y,
    pch, # Point character type
    cex, # Character expansion - i.e size
    col, # Colour of the point
    bg # Background colour or point
    )
```

Example:

```
> plot(c(0,2),c(0,25), type='n')
> points(x=rep(1,25), y=1:25, pch=1:25)
```



Illustration 29: Plot - points

### 13.1.8 Symbols

symbols(): Insert a shape at any position of a currently open plot.

```
symbols(x, y, # x,y plot co-ordinates
    circles, # Draw circle with of diameter (e.g. circle=0.8)
    squares, # Draw square of side length (e.g. square=0.5)
    rectangles, # Draw rectangle (side lengths specified by matrix)
    stars, # Draw star (points etc specified by 3 column matrix)
    thermometers, # Craw with matrix of 3 or 4 columns
    boxplots, # Draw with matrix of 5 columns
    add=T, # Adds the symbol to the current plot
    bg, fg, lwd... # General settings - see help(symbols)
)
```

Example:

```
> plot(c(0,4),c(0,6), type='n')
> symbols(x=1, y=3, circles=3,
        bg='red', fg='green',
        lwd=3, add=T
    )
```



Illustration 30: Plot - symbols

### 13.1.9 Segments

segments(): Insert line segment at any position of a currently open plot.

```
> segments(x0, y0, # x,y plot co-ordinates for the start of the segment
    x1, y1, # x,y plot co-ordinates for the end of the segment
    col, # Colour of the line = 1:8
    lty, # Line type = 1:6 (dashed, dotted, whole etc)
    lwd # Line width
)
```

Example:

```
> plot(c(0,4),c(0,6), type='n')
> segments(x0=rep(1,6), y0=1:6,
        x1=rep(3,6), y1=1:6,
        lty=1:25
        )
```



Illustration 31: Plot - lines

### 13.1.10 Polygon

polygon(): Insert any shape at any position of a currently open plot.

```
    > polygon(x,
the shape
    y, # Vector of corresponding y co-ordinates for the shape (matched
to x)
    density, # Density of shading lines that fill the object
    angle, # Slope of shading lines that fill the object
    col, # Colour of shading lines (if density=NA then col is fill colour)
    border, # Colour of the border
    lwd... # Line width
)
```

Example:

```
> plot(c(0,8),c(0,3), type='n')
> polygon(c(1,1:7,7), c(0,1,2,1,2,1,2,1,0), col='blue', lwd=3)
> polygon(c(1,7,7), c(0,0,1), col='yellow', angle=90, density=7)
```



Illustration 32: Plot - polygons

### 13.1.11 Arrows

arrows(): Draw an arrow between pairs of coordinates in a current open plot.
> arrows(x0, y0,
\# Co-ordinates of points from which to draw
x1, y1, \# Co-ordinates of points to which to draw
length, \# Length of edge of arrow head
angle, \# Angle from arrow shaft to arrow head
code, \# 1:3 specifying the type of arrow head
col, lty, lwd... \# Other options
)

Example:

```
> plot(c(0,8),c(0,3), type='n')
> arrows(x0=1,y0=1,x1=6,y1=2,col='dark green',lty=2, lwd=3)
```



Illustration 33: Plot - arrows
13.1.12 Lines

- lines(): A generic function taking coordinates given in various ways and joining the corresponding points with line segments.
- abline(): This function adds one or more straight lines through the current plot. It can specify intercept and slope, horizontal or vertical and can take intercept \& slope from an Im object.
- curve() Draws a curve corresponding to a function over the interval from, to. curve() can plot also an expression in the variable xname, default $x$. Can add the line of an equation to a plot. Note: must set add=TRUE.
Example:

```
> plot(c(0,8),c(0,8), type='n')
> lines(x=c(1,5,8), y=c(2,3,1), col='blue', lwd=3)
> x=c(1,2,3,4,5); y=c(2,2,1,1,0)
> abline(lm(y~x))
> curve(x+0.1*x^2, add=T)
```



Illustration 34: Plot - lines, curves

```
Example: abline()
    > plot(1:21,-10:10)
    > abline(h=0, lty=2) # Put horizontal line at y=0
    > abline(v=5, lty=1) # Put vertical line at x=5
```



Illustration 35: Plot - abline

### 13.1.13 Identity

Identify and label a point on a scatter-plot. Use the cursor over a point and it will identify it.

```
> x=rnorm(100,0,2)
> y=rnorm(100,3,3)
> plot(x,y)
> identify(x, y, plot=TRUE)
```



Illustration 36: Plot - identity

### 13.1.14 Adding equations / Greek letters to graphs

expression(): Creates or tests for objects of mode expression.
examples:
$>\operatorname{plot}(c(0,10), c(0,10)$, type='n')
$>\operatorname{text}(x=5, y=8$, expression(lambda $==1.3)$ )
$>\operatorname{text}(x=5, y=6$, expression(bar(X)[female]==0.55))
$>\operatorname{text}\left(x=5, y=4\right.$, expression $\left.\left(y[3] \sim x^{\wedge} 2 \sim m^{\wedge}-2\right)\right)$


Illustration 37: Plot - expression

### 13.2 Boxplots

A box plot or boxplot is a method of representing statistical data on a plot. It consists of a rectangle drawn to represent the second and third quartiles, usually with a vertical line inside to indicate the median value. The lower and upper quartiles are shown as horizontal lines either side of the rectangle.


Illustration 38: Boxplot

Figure 37: Plot - boxplot
Considering the following example. As can be seen from the summary(v) the minimum and maximum points are marked at 5 and 425 . The median of 152 is flanked by the lower quartile at 69.5 and the upper quartile at 272.5.

```
> v = c(101, 111, 112, 123,141, 152,193,141,
    51,19,43,74,45,26,83,42,65,32,5,
    322, 354, 385, 377, 381, 314, 425,416,
    214,233,234, 226,237, 248,269,276
    )
```

> summary(v)
Min. 1st Qu. Median
$5.0 \quad 69.5 \quad 152.0$
Mean 3rd Qu
Max.
$184.9 \quad 272.5425 .0$


Illustration 39: Boxplot 2
> boxplot(v)


Illustration 40: Boxplot 3

### 13.3 Saving

### 13.3.1 Data

Data can be saved in a number of formats, in this case it is saved in .csv formatted file. In the example below the plot is saved to: 28-Sep-2018_00.38-graph_name.csv.

```
> logdir = '/var/log/R/' # Make sure directory has +w rights
> filename = 'graph_name' # Make sure directory has +w rights
# Export output
> writeto = paste0(logdir,"/",
    format(Sys.time(), "%d-%b-%Y_%H.%M-"),
    filename, ".csv"
    )
```

> write.csv(df.normal, writeto)

### 13.3.2 Plots

Plots can also be saved from graphical format to file. In the example below the plot is saved as a .png to: 28-Sep-2018_00.35-graph_name.png. The graphic can also be saved in formats like .pdf.

```
logdir = '/var/log/R/' # Make sure directory has +w rights
filename = 'graph_name' # Make sure directory has +w rights
# Export output
> writeto = paste0(logdir,"/",
                            format(Sys.time(), "%d-%b-%Y_%H.%M-"),
    filename, ".png"
    )
> dev.copy(png, writeto) # copies the graphics contents
> dev.off() # Shutdown graphic to push to file.
```


### 13.4 Exercise 1: Making a mess

```
> plot(x=1:100,y=rnorm(100))
```



Illustration 41: Plot exercise - start
Taking Illustration 41 apply graphical features liberally to make it as ugly as possible © .
Adjust the default background colour.

```
> par(bg = "green") # Set background to green
> plot(x=1:100,y=rnorm(100),
    pch = 13, # Change icons to triangles
    cex = 2, # Enlarge icon to 200%
    lwd = 3, # Change line width
    col="orange", # Icon colour
    bg="yellow", # Icon background colour
    col.main = "orange", # Change title colour
    col.axis = "red", # Change axis colour
    col.lab = "white", # Change label colour
    font.axis = 3, # Change axis font
    font.lab = 4, # Change label font
    family = "sans", # Change font
    )
```

Locate a point on the plot with the locator() function. Run the function and click on the plot, it will return the location of the cursor. For the text line the shell will hold until the point is clicked.

```
> locator(1)
$x
[1] -16.85286
$y
[1] 3.984065
```

```
> text(locator(1), "A bit of text is here")
> lines(locator(2),col="white",lty=1,lwd=3) # Draw a line
> lines(locator(2),col="black",lty=1,lwd=3) # Draw a line
> locator(2)
$x
[1] 52.10779 51.52666
$y
[1] 2.06091 -2.54633
> points(x=rep(1,25),y=1:25, type= 'n')
# Draw a line
> segments(52.10779, 2.06091, 51.52666, -2.54633,col="red",lty=2,lwd=4)
# Draw a line
> lines(c(4.455207, 96.273600), c(-0.33893, -0.30497) ,col="yellow",lty=2,lwd=4)
> title("Is it Saint Patricks Day") # Add a title
> polygon(c(1,1:7,7), # Add a polygon
        c(0,1,2,1,2,1, 2,1,0),
        col='blue', lwd=3
        )
```



Illustration 42: Plot - mess

### 13.5 Exercise 2: Create a boxplot

As has been demonstrated in section 13.2 Boxplots $R$ comes with a boxplot() function as in Illustration 43. Assuming there is no such function create a boxplot for the output of the function rnorm $(100,5,1.5)$ using the other tools available.

```
> a = rnorm(100,5,1.5)
> boxplot(a)
```



Illustration 43: Boxplot exercise

Answer:

```
> a = rnorm(100,5,1.5)
mean_a = mean(a)
> sd_a = sd(a)
min_a = min(a)
> max_a = max(a)
> plot(c(0, 0, min_a, min_a), # Plot area
        c(0,max_a,max_a,0),
        ylim = c(0,10), xaxt = 'n', yaxt = 'n',
        type = 'n', xlab = '', ylab = ''
    )
> segments(1, min_a, # Dotted line line
        1, max_a,
        col="black",lty=2,lwd=1
        )
> segments(0.9,max_a, # Top line
        1.1, max_a,
        col="black",lwd=1
        )
> segments(0.9,min_a, # Bottom line
        1.1,min_a,
        col="black",lwd=1
        )
> polygon(c(0.8, 1.2, 1.2, 0.8), # Draw polygon
        c(mean_a - sd_a,mean_a - sd_a, mean_a + sd_a,mean_a + sd_a),
        border = "black",
        col = "white",
        lwd=1
        )
> segments(0.8,mean_a, 1.2, mean_a, # Draw middle line
        col="black",lwd=4
        )
> axis(2, at=c(1,2,3,4,5,6,7,8), labels=c(1,2,3,4,5,6,7,8))
```



Illustration 44: Boxplot answer

### 13.6 Multiple graphs

### 13.6.1 $\operatorname{par}()$ function again

With the $\operatorname{par}($ ) function, the options mfrow=c(nrows, ncols) can be included to create a matrix of nrows x ncols plots that are filled in by row. mfcol=c(nrows, ncols) fills in the matrix by columns. To plot the model model. 1 in the example will shuffle through four plots.

- Residuals vs Fitted
- Normal Q-Q
- Scale-Location
- Residuals vs Leverage

```
> x = c(1,2,2,3,2,3,4,3,4,5,3)
>y = c(4,8,6,3,5,7,9,2,1,7,4)
> model.1 = lm(y ~ x)
> plot(model.1)
```

Hit <Return> to see next plot:
Hit <Return> to see next plot:
Hit <Return> to see next plot:
Hit <Return> to see next plot:

However by adjusting the graphical parameters it is possible to create a top-level graphical plot area that will include the four plots on the one main plot area. As there are four plots then a $2 \times 2$ main plot frame is required.

```
> x = c(1,2,2,3,2,3,4,3,4,5,3)
>y=c(4,8,6,3,5,7,9,2,1,7,4)
> model.1 = lm(y ~ x)
> par(mfrow=c(2,2)) # Adding this line creates the top-level plot area
> plot(model.1)
```



## Illustration 45: Multiple graphs

### 13.7 Exercise 3a: Setting graph parameters

Using these data, create a publication quality graph like this.
HINT: to get the axis tick marks on the inside use the tck argument in your plot call (e.g. tck=0.03).
$>x=1: 20$
$>y=\operatorname{seq}($ from=3, to=7, length.out=20)+rnorm(20, 0,2)

Answer:

```
> x = 1:20
> y = seq(from=3, to=7, length.out=20)+rnorm(20,0,2)
> plot(x,y, type='n',
    xlab = 'Years since establishment',
        ylab = 'Population size (x 1000)',
        xaxt = 'n', yaxt = 'n',
        bty = 'l',
        xlim = c(0,20),
        ylim = c(0,10),
        font.lab=2
    )
> points(x,y, pch=21, bg="black", cex=2)
> axis(1, seq(0,20,5),las=2, tck = 0.02)
> axis(2, seq(0,10,2), tck = 0.02)
```



Illustration 46: Exercise - setting graph parameters

### 13.8 Exercise 3b: Setting graph parameters

Add a second plot using these additional data.

```
> x = 1:20
> proportion.females = runif(20,0.3,0.7)
> y = proportion.females
> plot(x,y, type='l',
    xlab = 'Years since establishment',
    ylab = 'Proportion of females in the population',
    xaxt = 'n', yaxt = 'n',
    bty = 'l',
    lty = 2,
    xlim = c(0,20),
    ylim = c(0,1),
    font.lab=2
        )
> axis(1, seq(0,20,5),las=2, tck = 0.02)
> axis(2, seq(0,1,0.2),tck = 0.02)
> text(x=1, y=0.98, "b", cex=1.5)
> text(x=10, y=0.15, expression(bar(X)[female]==0.55), cex=1.3)
```



Illustration 47: Exercise - setting graph parameters 2

Create a multiple frame to hold both graphs.
mtext(): Write Text into the Margins of a Plot.

```
# Data to be plotted
> x = 1:20
> y = seq(from=3, to=7, length.out=20)+rnorm(20,0,2)
> z = runif(20,0.3,0.7)
# First graphic area (left side)
> par(fig=c(0,0.5,0,1))
# Plot the points on first graph
> plot(x,y, type='n',
    xlab = '',
    ylab = 'Population size (x 1000)',
    xaxt = 'n', yaxt = 'n',
    bty = 'l',
    xlim = c(0,20),
    ylim = c(0,10),
    font.lab=2
    )
# Redefine the points
> points(x,y, pch=21, bg="black", cex=2)
# Add the first graph axis
> axis(1, seq(0,20,5),tck = 0.02)
> axis(2, seq(0,10,2), tck = 0.02)
# Add the "a" and the function term to 1st graph
> text(x=1, y=9.8, "a", cex=1.5)
> text(x=10, y=9, expression(lambda == 1.3), cex=1.5)
# Second graphic area (right side)
> par(fig=c(0.5,1,0,1), new=T)
# Plot the dashed line
> plot(x,z, type='l',
    xlab = '',
    ylab = 'Proportion of females in the population',
    xaxt = 'n', yaxt = 'n',
    bty = 'l',
    lty = 2,
    xlim = c(0,20),
    ylim = c(0,1),
    font.lab=2
    )
# Add in the axis graphs
> axis(1, seq(0,20,5),tck = 0.02)
> axis(2, seq(0,1,0.2),tck = 0.02)
# Add the "b" and the function term to 2nd graph
> text(x=1, y=0.98, "b", cex=1.5)
> text(x=10, y=0.15, expression(bar(X)[female]==0.55), cex=1.3)
# Add the common text under both graphs
> mtext("Years since establishment", side = 1, line = -2, outer = TRUE, cex=1.3)
```



Illustration 48: Exercise - setting graph parameters 3

### 13.9 Exercise 4a: Prediction plots

Using the model Im(begging~sex+food+sex*food, data = owl)
Extract predictions (with their standard errors) for both males and females, for a range of food values from 20-30 using the predict() function.

```
# Add libraries
> library(scales)
# Import the data
> owl = read.csv('owl_data.csv')
> owl.lm = lm(begging ~ sex + food + sex * food, data = owl)
> male_predict = data.frame(sex=rep('Male',11), food=10:20)
> female_predict = data.frame(sex=rep('Female',11), food=10:20)
> male_out = predict.lm(owl.lm, male_predict, se.fit=T)
> female_out = predict.lm(owl.lm,female_predict, se.fit=T)
# Plot 1
# Generate the first plot
> plot(x=seq(10,20),y=male_out$fit,type='n',
    xlab='Time since last meal',
    ylab='Begging rate',
        bty='l', ylim=c(8,25), cex.lab=1.2
    )
# Add lines to plot
> lines(x=seq(10,20),male_out$fit,
        type='b', pch=16, col='blue'
        )
> lines(x=seq(10,20),female_out$fit,
        type='b', pch=16, col='red'
        )
# Plot 2: Add Standard Errors (SE) lines on the plot
# Add SE lines for males
> lines(x=seq(10,20),male_out$fit+male_out$se.fit,
        type='l', col='blue', lty=2
        )
> lines(x=seq(10,20),male_out$fit-male_out$se.fit,
        type='l', col='blue', lty=2
        )
# Add SE lines for females
> lines(x=seq(10,20),female_out$fit+female_out$se.fit,
        type='l', col='red', lty=2
        )
> lines(x=seq(10,20),female_out$fit-female_out$se.fit,
        type='l', col='red', lty=2
        )
# Plot 3: now add SE polygons
# Add SEs for males
> polygon(x=c(10:20, 20:10),
        y=c(male_out$fit+male_out$se.fit,
        rev(male_out$fit-male_out$se.fit)),
        col=alpha("blue", 0.3), border="blue"
```

```
    )
# Add SEs for females
> polygon(x=c(10:20, 20:10),
    y=c(female_out$fit+female_out$se.fit,
    rev(female_out$fit-female_out$se.fit)),
    col=alpha("red", 0.3), border="red"
    )
# Add text to the plot
> text(16,19,'Male', cex=1.5, col='blue')
> text(15,9,'Female', cex=1.5, col='red')
```



Illustration 49: Exercise - prediction plots

### 13.10 Exercise 4b: Prediction plots

Modify the previous model to a Poisson distribution.

```
glm(begging \(\sim s e x+f o o d+s e x * f o o d\), data=owl, family=poisson)
```

Repeat exercise 4 a , but using the predictions from the model based on a poisson distribution.

HINT:check if your predictions are at the log scale and need to be exponentiated. check the predict function argument type. This needs to be set to response.

```
# Add libraries
> library(scales)
# Import the data
> owl = read.csv('owl_data.csv')
> owl.lm = glm(begging ~ sex+food+sex*food, data=owl, family=poisson)
> male_predict = data.frame(sex=rep('Male',11), food=10:20)
> female_predict = data.frame(sex=rep('Female',11), food=10:20)
> female_out = predict.glm(owl.lm, male_predict, se.fit=T, type="response")
> male_out = predict.glm(owl.lm,female_predict, se.fit=T, type="response")
# Plot 1
# Generate the first plot
> plot(x=seq(10,20),y=male_out$fit,type='n',
    xlab='Time since last meal',
    ylab='Begging rate',
    bty='l', ylim=c(0,60), cex.lab=1.2
    )
# Add lines to plot
> lines(x=seq(10,20),male_out$fit,
        type='b', pch=16, col='blue'
        )
> lines(x=seq(10,20),female_out$fit,
    type='b', pch=16, col='red'
    )
# Plot 2: Add Standard Errors (SE) lines on the plot
# Add SE lines for males
> lines(x=seq(10,20),male_out$fit+male_out$se.fit,
        type='l', col='blue', lty=2
    )
> lines(x=seq(10,20),male_out$fit-male_out$se.fit,
    type='l', col='blue', lty=2
    )
# Add SE lines for females
> lines(x=seq(10,20),female_out$fit+female_out$se.fit,
    type='l', col='red', lty=2
    )
> lines(x=seq(10,20),female_out$fit-female_out$se.fit,
    type='l', col='red', lty=2
    )
```

```
# Plot 3: now add SE polygons
# Add SEs for males
> polygon(x=c(10:20, 20:10),
    y=c(male_out$fit+male_out$se.fit,
    rev(male_out$fit-male_out$se.fit)),
    col=alpha("blue", 0.3), border="blue"
        )
# Add SEs for females
> polygon(x=c(10:20, 20:10),
    y=c(female_out$fit+female_out$se.fit,
    rev(female_out$fit-female_out$se.fit)),
    col=alpha("red", 0.3), border="red"
    )
# Add text to the plot
> text(16,40,'Female', cex=1.5, col='red')
> text(15,9,'Male', cex=1.5, col='blue')
```



Illustration 50: Exercise - prediction plots 2

### 13.11 Exercise 5: Visualising plot data

Import the bird_egg.csv file.
Plot the relationship between eggs (y) and age (x) for clutch 1 (blue points) \& clutch 2 (red points) and save it as a pdf file.
HINT: jitter() could help visualise the points here eggs is the response variable (y) and age the explanatory ( x ).

EXTRA: extract predictions from a model where age and clutch (and their interaction) explain the number of eggs. Plot these predictions with their standard errors over the scatter-plot above.

```
> install.packages('scales')
# Add libraries
> library(scales)
# Import the data
> bird = read.csv('bird_egg.csv')
> names(bird)
#[1] 'individual' 'year' 'clutch' 'age' 'eggs' 'dist_food'
#[7] 'fail_fledge'
# Plot the points and
> x1 = bird[bird[,3]==1,4] # Plot the 'age' for the 1st clutch
> y1 = bird[bird[,3]==1,5] # Plot the 'eggs' for the 1st clutch
> x2 = bird[bird[,3]==2,4] # Plot the 'age' for the 2nd clutch
> y2 = bird[bird[,3]==2,5] # Plot the 'eggs' for the 2nd clutch
# Plot the eggs as a DV for age as the IV for 1st clutch
> plot(x1,y1,col='blue',
    xlab='Age', ylab='Eggs'
        )
# Plot the eggs as a DV for age as the IV for 2nd clutch
> points(x2,y2, col='red')
# Add a small amount of noise to the vector.
> points(jitter(x1),jitter(y1), col='blue')
> points(jitter(x2),jitter(y2), col='red')
# Get data for clutch 1 & 2 (poisson)
> bird1 = bird[bird[,3] < 3,]
> mod1 = glm(eggs~age * as.factor(clutch),
        data=bird1, family='poisson'
    )
> summary(mod1)
# Extract dataframe for 'age' and 'clutch'
> new_data1 = data.frame(age=1:10, clutch=rep(1,10))
> new_data2 = data.frame(age=1:10, clutch=rep(2,10))
# Get predictions
> pred_clutch1 = predict.glm(mod1, new_data1, se.fit=T, type='response')
> pred_clutch2 = predict.glm(mod1, new_data2, se.fit=T, type='response')
# Add clutch1 prediction
# Add lines
> lines(x = 1:10, y = pred_clutch1$fit,
    lty = 1, lwd = 2, col = 'blue'
```

```
    )
# Add blue filled in pologon area
> polygon(x = c(1:10, 10:1),
    y = c(pred_clutch1$fit+pred_clutch1$se.fit,
    rev(pred_clutch1$fit-pred_clutch1$se.fit)),
    col = alpha('blue',0.3), border='blue'
    )
# Add clutch2 prediction
# Add lines
> lines(x = 1:10, y = pred_clutch2$fit,
    lty = 1, lwd = 2, col = 'red'
    )
# Add red filled in pologon area
> polygon(x = c(1:10, 10:1),
    y = c(pred_clutch2$fit+pred_clutch2$se.fit,
        rev(pred_clutch2$fit-pred_clutch2$se.fit)),
        col = alpha('red',0.3), border='red'
        )
}
# Add labels
> text(x = 6, y = 5.5, 'Clutch 1',
    cex = 1.5, col = 'blue'
        )
> text(x = 6, y = 2, 'Clutch 2',
    cex = 1.5, col = 'red'
        )
```



Illustration 51: Exercise - visualising plot data

### 13.12 Exercise 6: Pretty plot

From the data $x y$ _data_quadplotex.csv produce a graph. The model is $y \sim x+x 2$.
HINT: Get the fitted values for the model prediction.

```
# Look at the data
> read.csv('xy_data_quadplotex.csv')
    x y
        3.9257408
        1.9763139
        3.1513985
        2.6076583
        5.0019918
        5.7526961
        6.3944896
        8.4130619
        9.4346628
10 10 9.7635863
11 11 8.9396002
12 12 7.2992594
1313 9.1645094
14 14 8.1535297
15 15 7.9302138
16 16 9.4534872
17 17 6.2583997
18 18 6.5865369
19 19 5.7238197
20 20 6.0263360
2121 3.0479905
22 22 3.3684445
23 23 4.5477625
24 24 3.1443438
25 25-0.3731705
26 26 0.9693107
27 27-5.7287515
28 28-3.4355719
29 29-2.9714692
30 30-7.3180748
# Import the data
> xy = read.csv('xy_data_quadplotex.csv')
# Create the initial plot with values
> plot(x,y)
# Create linear model
> mod.1 = lm(xy$y ~ xy$x + I(xy$x^2))
> y.fit = fitted(mod.1)
# Draw the fit line on plot
> lines(xy$x,y.fit, lwd=2)
> get.y = matrix(c(xy$x,y.fit), ncol=2)
# Show area under curve for range 1 - 7
x1.range <- 1:7
y1.range <- get.y[get.y[,1]>=1 & get.y[,1]<=7,2]
# Show area under curve for range 7 - 20
> x2.range <- 7:20
y2.range <- get.y[get.y[,1]>=7 & get.y[,1]<<20,2]
# Show area under curve for range 20 - 30
x3.range <- 20:30
```

```
> y3.range <- get.y[get.y[,1]>=20 & get.y[,1]<=30,2]
# Add shading area under the curve for x - 1 - 7
> polygon(c(1,x1.range,7),
    c(min(y),y1.range,min(y)), density=20,
    col="green", angle=45, border=NA
    )
# Add shading area under the curve for x - 7-20
> polygon(c(7,x2.range, 20),
        c(min(y),y2.range,min(y)), density=20,
        col="red", angle=45, border=NA
        )
    # Add shading area under the curve for x - 20 - 30
    > polygon(c(20,x3.range,30), c(min(y),y3.range,min(y)), density=20, col="yellow",
angle=45, border=NA)
    # Add labeling to each shaded area
> text(3.5,-3, "Treatment\nA", cex=1)
> text(12.5,-3, "Treatment\nB", cex=1)
> text(24,-3, "Treatment\nC", cex=1)
```



Illustration 52: Exercise - pretty plot

### 13.13 Exercise 7: Icon and colour table

```
# Add the plot
> plot(x = c(0,1.2),y = c(0,25),
    col = "white", xlab = "", ylab = ""
    xaxt = "n",yaxt = "n"
    )
# Add points for icons
> points(x = rep(0.1,25),
    y = 1:25,pch = 1:25,
    cex = 1.2
    )
# Add text for icons
> text(x = rep(0.1,25),
    y = 1:25,paste("pch",1:25),
    pos = 4, offset = 1
        )
# Add points for colour
> points(x = rep(0.4,8)
        y = 10:17,col = 1:8,
        pch = 16, cex = 3
        )
# Add text for colour
> text(x = rep(0.4,8),y = 10:17,
    paste("colour",1:8),pos = 4,
    offset = 1
        )
# Add lines
> segments(x0 = rep(0.7,20),y0 = 11:16,
    x1 = rep(0.9,20),lty = 1:6,
    lwd = 1.3,col = "black"
        )
# Add text to lines
> text(x = rep(0.9,20),y = 11:16,
    paste("lty",1:6),pos = 4,
    offset = 1
    )
# Add title text
> title_msg = "Exercise 7: reproduce & save this plot for future reference"
> mtext(title_msg,
    side = 3, outer = TRUE, line = -2.2:-2, font = 2
    )
```


## Exercise 7: reproduce \& save this plot for future reference



Illustration 53: Exercise - useful table

## 14. Generalised Linear Mixed Models (GLMM)

Taking the earlier prediction:

```
> owl = read.csv('owl_data.csv')
> owl.lm = lm(begging ~ sex + food + sex * food, data = owl)
```

The assumption was that the residuals were random and had no relationship with eachother. However it could be observed for example that there is a higher possibility of the residuals below the model line being say male in blue and above the line being female. Thus there is non-independence of the residuals and this needs to be accounted for. One way is to do separate ANCOVA for set, male and female.


However this could get impractical quite quickly. Imagine if there were samples taken from a number of sources and for practical reasons there were more samples taken from certain sources than others. Now there is non-independence between the samples taken from each source and because of the different ratios of samples from each it must be accounted for in the linear models.

- nlme: Non-linear Mixed-Effects Models (NLME). This generic function fits a non-linear mixed-effects model in the formulation described in Lindstrom and Bates (1990) but allowing for nested random effects. The within-group errors are allowed to be correlated and/or have unequal variances.
- nlme: Fit Linear Mixed-Effects Models (LMM). Fit a LMM to data, via the Restricted (or Residual, or Reduced) Maximum Likelihood (REML).

Import the owl_data.csv file. There are four models. The first two are standard Linear Models. The second two models fit the same model as the $\operatorname{Im}()$ function, however they must have at least one random effect, in this case the (1/nest) argument. If you want to fit another type of distribution you use $g / m e r()$ and set the family argument. It is demonstrated below for the family poisson however in this case it would fail to converge as the dataset is not suitable for that distribution.


Illustration 55: Generalised Mixed Models

```
# Install packages (Unhash the first time to install packages)
# install.packages('nlme')
# install.packages('lme4')
# Load libraries
> library(nlme)
> library(lme4)
# Generate models
> mod.lm = lm(begging ~ food + sex + food * sex + I(food^2), data = owl)
> mod.glm = glm(owl.lm = lm(begging ~ sex + food + sex * food, data = owl)(food^2),
    data=owls, family=gaussian
    )
> mod.lme = lme(owl.lm = lm(begging ~ sex + food + sex * food, data = owl)(food^2),
    random =~ 1|nest, data = owl
        )
> mod.lmer = lmer(begging ~ food + sex + food * sex + I(food^2) + (1|nest),
    data = owl
    )
> mod.glmer = glmer(begging ~ food + sex + food * sex + I(food^2) + (1|nest),
    data=owl, family=poisson
    )
> par(mfrow=c(2,2))
> plot(mod.lm)
```



Illustration 56: Plot - Fitting Linear Models
> $\operatorname{par}(m f r o w=c(2,2))$
> plot(mod.glm)


Illustration 57: Plot - Fitting Generalised Linear Models
> plot(mod.lme)


Illustration 58: Plot - Linear Mixed-Effects Models


Illustration 59: Plot - Fit Linear Mixed-Effects Models

## 15. Qualitative Data Analysis with $R$

### 15.1 Introduction

As has already been shown the $R$ programming language is very powerful for quantitative analysis, but what or Qualitative analysis? $R$ has a R Qualitative Data Analysis (RQDA) for qualitative text and PDF document analysis.

It is particularly useful for inductive thematic analysis however for deductive analysis it is necessary to upload Categories and Codes one by one. RQDA Code Builder resolves this.
This document demonstrates how to use RQDA() and the RQDA Code Builder on a GNU/Linux platform. $R$ and RQDA() can be used on other platforms like Microsoft Windows and as the RQDA Code Builder is Python3 based it can easily be adapted for other platform implementations.
It is necessary to have python3 installed on the platform. Use the Software Manager for your GNU/Linux flavour or install using apt from the shell terminal.

```
$ sudo apt install python3
```

\$ sudo apt-get install python-yaml

Confirm the install and the version of python3.
\$ python3 --version
Python 3.5.2

### 15.2 Qualitative Content Analysis

Qualitative Content Analysis follows a procedure (Flick, 2014):

1. Deciding the research question
2. Selecting material
3. Building a coding frame
4. Segmentation
5. Trial coding
6. Evaluating and modifying the coding frame
7. Main analysis
8. Presenting and interpreting the findings.

### 15.3 Coding

Assuming that steps 1 and 2 are completed and the next step is the building of a coding frame. There are two approaches, inductive and deductive.

The inductive approach has codes extracted directly from the source data. As the researcher reads through each source file (interviews, papers, etc..), he or she highlights key lines and creates a code for it. These codes are added and modified as the researcher reads through all the source material. The codes are then bundled into codes of common category. RDQA() is very suitable for this approach.
The deductive approach involves the researcher developing codes and categories in advance, in a scheme. These codes are then applied to the source data. As RDQA()
expects codes to be added one by one through the graphical interface this is difficult. Application of the rqda_code_builder.py program described here helps to fix this.

### 15.4 Starting RQDA()

Create a directory as a parent for the project and open a command shell in it. Within the parent directory create a Sources directory. Place the source files in the Sources directory. In this example you can see two source files but typically this would be many files associated with interviews, observation logs, etc..

```
$ mkdir Sources
```



## Sources

## Illustration 60: Sources directory

\$ ls Sources
Colours_of_Health_and_Sickness_Sociocult.txt
Psychological_Properties_Of_Colours.txt

Run the ' R ' program.

```
$ R --quiet
```


### 15.4.1 Add the RQDA library

Add the $R D Q A()$ library, this is the program that allows the researcher to analyse the data.

```
> library(RQDA)
Loading required package: RSQLite
Loading required package: gWidgetsRGtk2
Loading required package: RGtk2
Loading required package: gWidgets
Loading required package: cairoDevice
Loading required package: DBI
Use 'RQDA()' to start the programme.
```

The graphical tool starts.


### 15.4.2 Create a Project

In the Graphical User Interface (GUI):

- Click New Project.
- Enter a name in the desired path: Colour_project.rqda. - Click OK.

A new project file appears in the directory.


Illustration 62: Project SQLite database

You may also notice in the $R$ shell that the following command is executed.
> [1] "~/Colour_project.rqda"

## Name the coder

Select the Settings tab and define the Name of Coder in the first box.


Illustration 63: RQDA() interface

### 15.4.3 Import source files to the project

The next step is to import source data. This can be achieved either through the GUI one by one, or in bulk using the $R$ function write.FileList() in the $R$ shell.

## Using the GUI

To use the GUI, select:

- The Files tab followed by the Import button.
- Browse to each file in turn and select.


Illustration 64: RQDA() files

## Using the R shell

An alternative mechanism is to use the $R$ shell. This command using the addFilesFromDir() function selects the files in the Sources directory that match the pattern. In this case all files that end in the pattern .txt.
Execute the command:

```
> addFilesFromDir('Sources', pattern = "*.txt$")
```

If you now check the GUI by clicking the Files tab, you will notice that the files from the Sources directory have been imported. Alternatively use the getFiles() function in the $R$ shell to confirm.

```
> getFiles()
[1] "Colours_of_Health_and_Sickness_Sociocult.txt"
[2] "Psychological_Properties_Of_Colours.txt"
attr(,"class")
[1] "RQDA.vector" "fileName"
```


### 15.5 Coding

### 15.5.1 Inductive approach

RQDA() is very suitable for the inductive approach however it takes significant time.


## Illustration 65: RQDA() files 2

Select each document in turn from the Files tab, a popup appears with the text from the source file selected.


Illustration 66: RQDA() Codes

On the main GUI click the Codes tab and as a line is read that requires coding select Add and create the code. For example, to add a code Black, click Add. Enter the new code in the box provided and click OK. With text highlighted, select the appropriate code, i.e. Black and click Mark.


Illustration 67: RQDA() Codes 2

As can be seen each line is tagged.

### 15.5.2 Deductive approach

Unfortunately there does not appear to be a mechanism to import codes into the $R Q D A()$ database in bulk. For the deductive approach a researcher may have tens or even hundreds of categories and codes, it could be necessary to bulk upload. Extract the files from the RQDA-Code-Builder.tgz archive file which will give all the files including the database from this example as well as the rqda_code_builder.py. Move this file to the parent directory of the Sources directory.


Sources


Colour_project.rqda

rqda_code_builder.py

## ABC

RQDA_codes.yaml

Illustration 68: RQDA() Code Builder
The RQDA Code Builder (rqda_code_builder.py) program resolves this.

## YAML file

YAML Ain't Markup Language (YAML) is a human-readable data serialisation language that is commonly used for configuration files, but can be used in many applications where data is being stored or transmitted. It is an ideal format for mapping of categories and codes.

The example project demonstrates how to deduct the following code schema as a YAML file in the same directory:

```
$ cat RQDA_codes.yaml
    RQDA_codes.yaml
    Colour:
        - 'Red'
        - 'Green'
        - 'Yellow'
        - 'Grey'
        - 'Black'
        - 'White'
        - 'Black'
        - 'Blue'
        - 'Pink'
        - 'Brown'
        - 'Purple'
        Psychological Properties:
            'Physical'
        - 'Intellectual'
        - 'Emotional'
        - 'Balance'
        - 'Spiritual'
        Floral Metaphors:
        - 'Daisy'
        - 'Juicy'
        - 'Apple'
        - 'Berry'
        - 'Flower'
        - 'Peach'
        Human Characteristics:
        - 'Divinity'
        - 'Eternity'
        - 'Infinity'
```


## Executing the RQDA Code Builder

Before executing the RQDA Code Builder it is important to shut down the RQDA() application by clicking on the $X$ in the top right corner and selecting $O K$ to the Really EXIT? question.


The file that the $R Q D A()$ program uses to store data is an SQLite database. It is the file that was created at the beginning when the project was opened (Colour_project.rqda). The RQDA Code Builder reads the YAML formatted schema and uploads it to the database. It also creates a Structured Query Language (SQL) log of each SQL command it executes and more importantly develops a set of $R$ commands that match text blocks to the codes. It has the following switches:

```
-c|--coder [Name] - Define coder, must match that from RQDA() settings.
-d|--database [DB] - Define path to SQLite3 database file.
-y|--yaml [YAML] - Define path to YAML code file.
```

Execute the command, check it is version 1.4 or greater and execute with the relevant switches as demonstrated.

```
$ cat rqda_code_builder.py | grep '# Version' | awk {'print $4'}
    1.4
$ ./rqda_code_builder.py -c JohnnyResearcher -d Colour_project.rqda -y RQDA_codes.yaml
RQDA Code Builder
Connecting to the SQLite3 database Colour_project.rqda.
Connected to the SQLite3 database Colour_project.rqda. Uploading..
Upload completed
A full list of SDL commands executed can be seen in the 'RQDA_SQL.log' file.
You can restart the RQDA() library with the following command in the R shell:
> RQDA()
```

Restart RQDA() as instructed.

```
> RQDA()
```

Two new files are created, RQDA_SQL.log which is a log of the SQL commands executed on the database as well as RQDA_R_search_cmds. $R$ which is a list of commands that will be executed in the $R$ shell to apply the deductive codes to the source files.


Illustration 70: RQDA() Search commands and Log files

## Applying the RQDA 'R' search commands

To apply the RQDA search commands execute the following command in the $R$ shell. This bulk executes the commands in the RQDA_R_search_cmds. $R$ file on all the source files.

```
> source('RQDA_R_search_cmds.R')
```


### 15.6 Reviewing Coding

Before diving into the coding within the various source files, review the coding statistics. It can be seen from this extract that 385 code blocks were applied to the source texts.


### 15.7 Reviewing the coded blocks

Selecting the Codes tab from the RQDA() GUI and select any particular code. In this case Brown was selected. Click on the Coding button and a popup appears with each instance of sentences within the source files where the word Brown or brown appeared, such sentences were tagged with the Brown tag. The popup also shows for each block the source file from where the sentence appeared.


Illustration 71: Reviewing the coded blocks

This performs an initial deductive coding. There may be quirks however, what if one interviewee kept referring to Beige but the researcher wanted to code it as Brown? or the researcher has a code Colour and some of the transcripts were transcribed in American English. In this case sentences with Color should be coded with Colour.
Carry out additional coding of sentences like this.
First find the CID of the Code for Brown. Select the Codes tab, click on the Brown code and its CID can be seen at the top of the pane as shown by the red circle in Illustration 72.


Execute the following two lines in the $R$ shell and they will be added to the main coding already performed.

```
> codingBySearch("Beige",fid=getFileIds(),cid=21,seperator="[.!?]")
> codingBySearch("beige",fid=getFileIds(),cid=21, seperator="[.!?]")
```


### 15.8 Visualising categories

There are some tools built into RQDA() for visualisation. For example using the D3.js JavaScript library for manipulating data. D3 helps bring data to life visually using Hypertext Markup Language (HTML), Scalable Vector Graphics (SVG), and Cascading Style Sheets (CSS).

### 15.8.1 Installing d3Network

On the $R$ Shell install D3.js and activate the d3Network within $R$.

```
> install.packages('d3Network')
> library(d3Network)
```


### 15.8.2 Visualising Categories



Illustration 73: Plot selected code categories with d3

Select the Categories tab, highlight a Category or many Categories using the ctrl button and right click. Scroll down to the Plot selected code categories with d3. A HTML page will pop-up with diagrams like these:


### 15.9 Summary

There are a lot more features to $R$ and $R Q D A()$ that can aid qualitative research. The additional RQDA Code Builder program (rqda_code_builder.py) allows the researcher to deductively pre-build a code schema and apply it automatically.

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